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Cartesian System of Rectangular Co-ordinates

- **Rectangular Co-ordinate Axes**

- **Distance Between two points**

The distance between two points

$P(x_1, y_1)$ and $Q(x_2, y_2)$ as $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The distance between a point p(x, y) from original (o, o) as

$OP = \sqrt{x^2 + y^2}$

Section Formula

(i) Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points on a line and R (x, y) divide P Q internally in the ration m and n, then the co-ordinate of R are

$$\left(\frac{mx_2 + n x_1}{m+n}, \frac{my_2 + n y_1}{m+n}\right)$$

(ii) If the point R (x, y) divide the line externally in the ratio m : n then co-ordinate of R as :

$$\left(\frac{mx_2 - n x_1}{m-n}, \frac{my_2 - n y_1}{m-n}\right)$$

(i) The co-ordinate of the mid-point of a line segment PQ as

$$\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$$

Area of a Triangle

Area of triangle

$$ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Co linearity of three points

$A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are three points are Collinear, If and only if the Area of the triangle ABC become Zero.

SLOPE of A Line

The Slope M of a line through

$A(x_1, y_1)$, and $B(x_2, y_2)$ is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

- Two line of Slopes m_1 and m_2 are parallel, if and only if $m_1 = m_2$
- Two line of Slopes m_1 and m_2 are perpendicular, if and only if $m_1 . m_2 = -1$

Angle between Two Lines

Let L1 & L_2 be two non-vertical and non-perpendicular lines with Slopes m_1 and m_2 respectively and be the angle between two lines, then

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 . m_2}\right|$, where

$$1 + m_1 m_2 = \theta$$

(i) If $\tan\theta$ is +ive, then angle is acute
(ii) If $\tan\theta$ is +ive, then angle is obtuse

Check Your Progress

1. Area of the triangle with vertices (4, 4); (3, –2) and (3, –16) is -
   (A) 7 (B) 18
   (C) 15 (D) 27
2. The Area of the triangle with vertices (1, 2);
   (5, 7) and (3, 8) is -
   a. (A) 6 (B) 7
   b. (C) 8 (D) 9
3. If (5, –4) and (–3, 2) are two opposite vertices of a square then its area is -
   (A) 50 (B) 75 (C) 25 (D) 100
4. The distance between feet of perpendiculars drawn from a point (–3, 4) on both axes is -
   (A) 5 (B) 2
   (C) 4 (D) 1
5. P,Q and R three points on the line joining A(–6, 8) and B(8, –6) such that AP = PQ = QR = RB, then coordinates of R are -
   (A) (–5/2, 9/2)
   (B) (5/2, 9/2)
   (C) (5/2, –9/2)
   (D) (9/2, –5/2)
6. The mid points of the sides of a triangle are (5,0), (5,12) and (0, 12) the orthocentre of this triangle is -
   a. (A) (0, 0) (B) (0, 24)
   b. (C) (10, 0) (D) $\left(\frac{13}{3}, 8\right)$

7. The extremities of hypotenuse of a right-angled triangle are (2, 0) and (0, 2), then locus of its third vertex is -

(A) $x^2 + y^2 - 2x - 2y = 0$

(B) $x^2 + y^2 + 2x - 2y = 0$

(C) $x^2 + y^2 - 2x + 2y = 0$

(D) $x^2 + y^2 + 2x + 2y = 0$

8. Line segment joining (5,0) and ($10cos\alpha$, $10sin\alpha$ is divided by a point P in ratio 2 : 3 . If α varies then locus of P is a -

(A) Pair of straight lines

(B) Circle

(C) Straight line

(D) Parabola

9. The distance between feet of perpendiculars drawn from a point (–3, 4) on both axes is -

(A) 5 (B) 2

(C) 4 (D) 1

10. P,Q and R three points on the line joining A(–6, 8) and B(8, –6) such that AP = PQ = QR = RB, then coordinates of R are -

(A) (–5/2, 9/2)

(B) (5/2, 9/2)

(C) (5/2, –9/2)

(D) (9/2, –5/2)

11. The mid points of the sides of a triangle are (5,0), (5,12) and (0, 12) the orthocentre of this triangle is -

(A) (0, 0) (B) (0, 24)

(C) (10, 0) (D) $\left(\frac{13}{3},8\right)$

12. The extremities of hypotenuse of a right-angled triangle are (2, 0) and (0, 2), then locus of its third vertex is -

(A) $x^2 + y^2 - 2x - 2y = 0$

(B) $x^2 + y^2 + 2x - 2y = 0$

(C) $x^2 + y^2 - 2x + 2y = 0$

(D) $x^2 + y^2 + 2x + 2y = 0$

13. Line segment joining (10,0) and ($20\cos\alpha$, $20\sin\alpha$) is divided by a point P in ratio 4 : 6. If α varies then locus of P is a -

(A) Pair of straight lines

(B) Circle

(C) Straight line

(D) Parabola

14. If (3, – 4) and (– 6, 5) are the extremities of the diagonal of a parallelogram and (–2,1) is its third vertex, then its fourth vertex is -

(A) (– 1, 0) (B) (– 1, 1)

(C) (0, – 1) (D) None of these

15. The coordinates of the point which divides the line segment joining (–3, –4)

and (–8, 7) externally in the ratio 7:5 are -

(A) (41/2, 69/2)
(B) (–41/2, –69/2)
(C) (–41/2, 69/2)
(D) None of these

16. The ratio in which the point (8, 4) divides the line segment joining the points (5, –2) and (9, 6) is -

(A) 2 : 1
(B) 3 : 1
(C) 2 : 3
(D) 1 : 2

Answer to check Progress

1 A 2 B 3 A 4 A 5D 6A 7 A 8 B 9C 110 C 11 D
12 A 13 B 14 A 15 C 16 B