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# CIRCLES

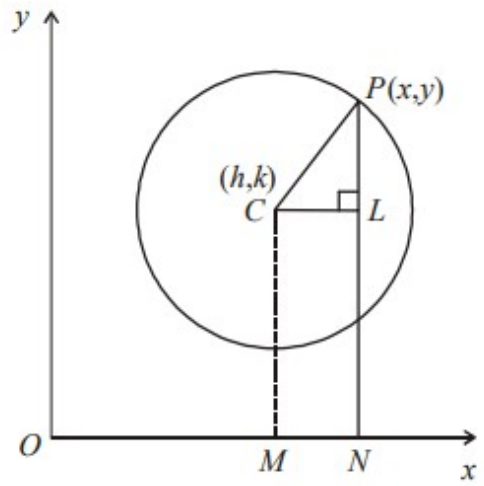
## Circle: -

A circle is the locus of points, which moves in a plane such that the distance from a fixed point in the same plane remains constant.

## Equation of a Circle

Let C be the centre and a be the radius of the circle. The co-ordinate of C (h, K), then

$$(x - h)^2 + (y - K)^2 = a^2$$




Let C be the centre and a be they radius of the circle. Coordinates of the centre are given to be (h, k), say. Take any point P(x, y) on the circle and draw perpendiculars CM and PN on OX. Again, draw CL perpendicular to PN.

In the right angled triangle CLP, $CL^2 + PL^2 = CP^2$

$$(x - h)^2 + (y - K)^2 = a^2$$

If the circle passing through origin then the reach of circle $h^2 + K^2 = a^2$

I. circle passes through the origin
$$h^2 + K^2 = a^2$$

II. circle does not pass through origin and the centre lies on the x-axis
$$(x - h)^2 + y^2 = a^2$$

III. circle passes through origin and the x-axis is a diameter
$$(x)^2 + y^2 \pm 2ax = 0$$

IV. centre of the circle is origin
$$(x)^2 + y^2 = a^2$$

V. circle touches the x-axis
$$(x)^2 + y^2 - 2hx - 2ay + h^2 = 0$$

VI. circle touches the y-axis
$$(x)^2 + y^2 - 2ax - 2ky + k^2 = 0$$

VII. circle touches both the axes
$$(x)^2 + y^2 - 2ax - 2ay + a^2 = 0$$

**General Equation of Circle in Second Degree in two variables**

The standard equation of a circle with centre

(h, k) and radius r is given by

$$(x - h)^2 + (y - K)^2 = r^2$$

$x^2 + y^2 + 2gx + 2fy + c = o$ is the general equation of circle

$centre = (-g, -f)$

$radius = \sqrt{g^2 + f^2 - C}$

## Check Your Progress

**Q.1** The length of the diameter of the circle

$x^2 + y^2 - 4x - 6y + 4 = 0$ is -

(A) 9 (B) 3 (C) 4 (D) 6

**Q.2** Which of the following is the equation of a circle?

(A) $x^2 + 2y^2 - x + 6 = 0$

(B) $x^2 - y^2 + x + y + 1 = 0$

(C) $x^2 + y^2 + xy + 1 = 0$

(D) $3(x^2 + y^2) + 5x + 1 = 0$

**Q.3** The equation of the circle passing through

(3, 6) and whose centre is (2, –1) is -

(A) $x^2 + y^2 - 4x + 2y = 45$

(B) $x^2 + y^2 - 4x - 2y + 45 = 0$

(C) $x^2 + y^2 + 4x - 2y = 45$

(D) $x^2 + y^2 - 4x + 2y + 45 = 0$

**Q.4** If (4, 3) and (–12, –1) are end points of a diameter of a circle, then the equation of the circle is-

(A) $x^2 + y^2 - 8x - 2y - 51 = 0$

(B) $x^2 + y^2 + 8x - 2y - 51 = 0$

(C) $x^2 + y^2 + 8x + 2y - 51 = 0$

(D) None of these

**Q.5** The radius of the circle passing through the points (0, 0), (1, 0) and (0, 1) is-

(A) 2 (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) $\frac{1}{2}$

**Q.6** The radius of a circle with centre (a, b) and passing through the centre of

the circle

$x^2 + y^2 - 2gx + f^2 = 0$ is-

(A) $\sqrt{(a-g)^2 + b^2}$ (B) $\sqrt{a^2 + (b+g)^2}$

(C) $\sqrt{a^2 + (b-g)^2}$ (D) $\sqrt{(a+g)^2 + b^2}$

**Q.7** If (x, 3) and (3, 5) are the extremities of a diameter of a circle with centre at (2, y). Then the value of x and y are-

(A) x = 1, y = 4 (B) x = 4, y = 1

(C) x = 8, y = 2 (D) None of these

**Q.8** If (0, 1) and (1, 1) are end points of a diameter of a circle, then its equation is-

(A) $x^2 + y^2 - x - 2y + 1 = 0$

(B) $x^2 + y^2 + x - 2y + 1 = 0$

(C) $x^2 + y^2 - x - 2y - 1 = 0$

(D) None of these

**Q.9** The coordinates of any point on the circle

$x^2 + y^2 = 4$ are-

(A) $(\cos \alpha, \sin \alpha)$

(B) $(4\cos \alpha, 4 \sin \alpha)$

(C) $(2\cos \alpha, 2\sin \alpha)$

(D) $(\sin \alpha, \cos \alpha)$

**Q.10** The parametric coordinates of any point on the circle $x^2 + y^2 - 4x - 4y = 0$ are-

(A) $(-2 + 2\cos \alpha, -2 + 2 \sin \alpha)$

(B) $(2 + 2\cos \alpha, 2 + 2 \sin \alpha)$

(C) $(2 + 2\sqrt{2} \cos \alpha, 2 + 2\sqrt{2} \sin \alpha)$

(D) None of these

**Q.11** The parametric coordinates of a point on the circle $x^2 + y^2 - 2x + 2y - 2 = 0$ are -

(A) $(1 - 2 \cos \alpha\ 1 - 2 \sin \alpha)$

(B) $(1+ 2 \cos \alpha, 1 + 2 \sin \alpha)$

(C) $(1+ 2 \cos \alpha, -1 + 2 \sin \alpha)$

(D) $(-1 + 2 \cos \alpha, 1 + 2 \sin \alpha)$

**Q.12** The equation $k(x^2 + y^2) - x - y + k = 0$ represents a real circle, if-

(A) $k < \sqrt{2}$ (B) $k > \sqrt{2}$

(C) $k > 1/\sqrt{2}$ (D) $0 < |k| \frac{1}{\sqrt{2}}$

**Q.13** If the equation

$px^2 + (2-q)xy + 3y^2 - 6qx + 30y + 6q = 0$ represents a circle, then the values of p and q are -

(A) 2, 2 (B) 3, 1

(C) 3, 2 (D) 3, 4

**Q.14** The circle represented by the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ will be a point circle, if-

(A) $g^2 + f^2 = c$ (B) $g^2 + f^2 + c = 0$

(C) $g^2 + f^2 > c$ (D) None of these

**Q.15** The equation of the circum-circle of the triangle formed by the lines $x = 0$, $y = 0$, $\frac{x}{a} - \frac{y}{b} = 1$, is -

(A) $x^2 + y^2 + ax - by = 0$

(B) $x^2 + y^2 - ax + by = 0$

(C) $x^2 + y^2 - ax - by = 0$

(D) $x^2 + y^2 + ax + by = 0$

## Stretch Yourself

1. Determine the value of k for the equation $k(x^2 + y^2) - x - y + k = 0$ represents a real circle.
2. Find that point where the line $x = 0$ touches the circle $x^2 + y^2 - 2x - 6y + 9 = 0$
3. Calculate the equation of the chord of contact of the circle $x^2 + y^2 + 4x + 6y - 12 = 0$ with respect to the point $(2, -3)$
4. The equation of the circle passing through the point (1, 1) and through the point of intersection of circles $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$
5. Calculate the length of tangent from the point (5, 1) to the circle $x^2 + y^2 + 6x - 4y - 3 = 0$

## Answer to check your progress

| | | | | |
|---|---|---|---|---|
| 1D | 2 D | 3 A | 4 B | 5 B |
| 6 A | 7 A | 8 A | 9C | 10 C |
| 11 C | 12 D | 13 C | 14 A | 15 B |