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Conic Section

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A conic section is the locus of a point P which moves so that its distance from a fixed point is always in a constant ratio to its perpendicular distance from a fixed line.

The fixed point is called the focus and is usually denoted by S.

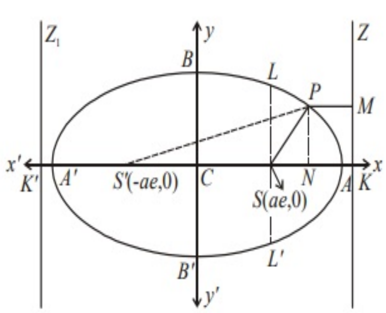
The fixed straight line is called the Directrix.

The straight line passing through the focus and perpendicular to the directrix is called the axis.

The constant ratio is called the eccentricity and is denoted by e.

ELLIPSE

"An ellipse is the locus of a point which moves in a plane such that its distance from a fixed point bears a constant ratio to its distance from a fixed line and this ratio is less than unity".



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Major axis	2a
Minor axis	2b
Principal axis	Major and Minor axis
Latus rectum	$\frac{2b^2}{a}$
Equation of directrix	$x = \pm\frac{a}{e}$
Eccentricity	$e^2 = 1 - \frac{b^2}{a^2}$

Parabola

"A parabola is the locus of a point which moves in a plane so that its distance from a fixed point in the plane is equal to its distance from a fixed line in the plane."

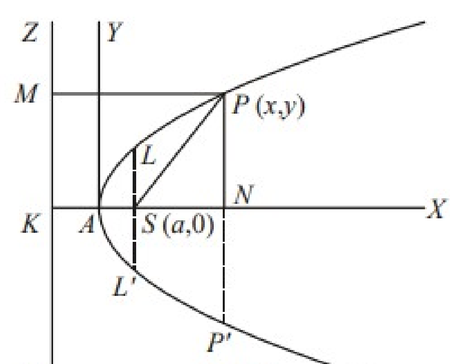
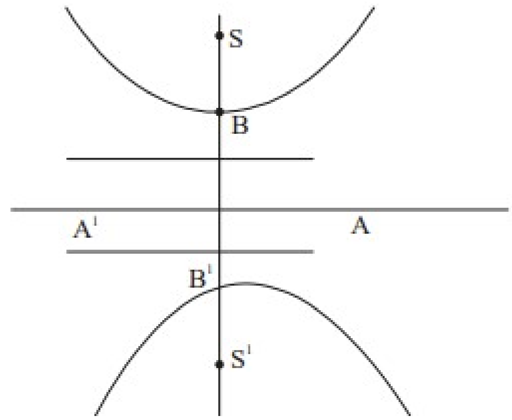


Fig.16.5

$$y^2 = 4ax$$

Forms	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Coordinates of vertex	(0,0)	(0,0)	(0,0)	(0,0)
Coordinates of focus	(a,0)	(-a,0)	(0,a)	(0,-a)
Coordinates of directrix	X =-a	X = a	Y =-a	Y =a
Coordinates of axis	Y =0	Y= 0	X = 0	X =0
Length of latus rectum	4a	4a	4a	4a



Hyperbola

Hyperbola is the conic in which eccentricity is greater than unity. The fixed point is called focus and the fixed straight line is called directrix.

Eccentricity(e)	$\sqrt{\frac{a^2 + b^2}{a^2}}$
Latus rectum	$\frac{2a^2}{b}$
directrix	y=$\pm\frac{b}{e}$
Length of transverse axis	2b
Length of conjugate axis	2a
vertices	$(0,\pm b)$
foci	$(0,\pm be)$
centre	(0,0)

Check Your Progress

1. The equation of the directrix of the parabola $x^2 = -8y$ is
   (A) x = 2 (B) y =2
   (C) y = –2 (D) x = –2

2. The equation to the parabola whose focus is (0, –3) and directrix y = 3 is
   (A) $x^2 = -12y$
   (B) $x^2 = 12y$
   (C) $y^2 = 12x$
   (D) $y^2 = -12x$

3. If (0, 0) be the vertex and 3x – 4y + 2 = 0 be the directrix of a parabola, then the length of its latus rectum is
   (A) 4/5 (B) 2/5

(C) 8/5 (D) 1/5

4. If $2x + y + \alpha = 0$ is a focal chord of the parabola $y^2 = -8x$, then the value of α is
   (A) –4 (B) 4
   (C) 2 (D) –2

5. The focal distance of a point(x_1, y_1) on the parabola $y^2 = 12x$ is
   (A) $x_1 + 3$ (B) $x_1 + 6$
   (C) $y_1 + 6$ (D) $y_1 + 3$

6. The equation to the ellipse (referred to its axes as the axes of x and y respectively) whose foci are (± 2, 0) and eccentricity 1/2, is-
   (A) $\frac{x^2}{12}+\frac{y^2}{16} = 1$
   (B) $\frac{x^2}{16}+\frac{y^2}{12} = 1$
   (C) $\frac{x^2}{16}+\frac{y^2}{8} = 1$
   (D) None of these

7. The eccentricity of the ellipse $9x^2 + 5y^2 - 30y = 0$ is-
   (A) 1/3 (B) 2/3
   (C) 3/4 (D) None of these

8. If the latus rectum of an ellipse be equal to half of its minor axis, then its eccentricity is-
   (A) 3/2 (B) $\sqrt{3}/2$
   (C) 2/3 (D) $\sqrt{2}/3$

9. If distance between the directrices be thrice the distance between the foci, then eccentricity of ellipse is-
   (A) 1/2 (B) 2/3
   (C) $\frac{1}{\sqrt{3}}$ (D) 4/5

10. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if-
   (A) $\Delta = 0, h^2 < ab$
   (B) $\Delta \neq 0, h^2 < ab$
   (C) $\Delta \neq 0, h^2 > ab$
   (D) $\Delta \neq 0, h^2 = ab$

11. The vertices of a hyperbola are at (0, 0) and (10, 0) and one of its foci is at (18, 0). The equation of the hyperbola is -
   (A) $\frac{x^2}{25}-\frac{y^2}{144}=1$
   (B) $\frac{(x-5)^2}{25}-\frac{y^2}{144}=1$
   (C) $\frac{x^2}{25}-\frac{(y-5)^2}{144}=1$

(D) $\frac{(x-5)^2}{25} - \frac{(y-5)^2}{144} = 1$

12. If the latus rectum of an hyperbola be 8 and eccentricity be $\frac{3}{\sqrt{5}}$, then the equation of the hyperbola is-
    (A) $4x^2 - 5y^2 = 100$
    (B) $5x^2 - 4y^2 = 100$
    (C) $4x^2 + 5y^2 = 100$
    (D) $5x^2 + 4y^2 = 100$

13. The foci of the hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$ are-
    (A) (2, 3), (5, 7) (B) (4, 1), (–6, 1)
    (C) (0, 0), (5, 3) (D) None of these

14. The foci of the hyperbola $4x^2 - 9y^2 - 36 = 0$ are-
    (A) $[\pm\sqrt{11}, 0]$ (B) $[\pm\sqrt{12}, 0]$
    (C) $[\pm\sqrt{13}, 0]$ (D) $[0, \pm\sqrt{12}]$

15. Foci of the hyperbola $\frac{x^2}{16} - \frac{(y-2)^2}{9} = 1$ are
    (A) (5, 2); (–5, 2)
    (B) (5, 2); (5, –2)
    (C) (5, 2); (–5, –2)
    (D) None of these

Stretch Yourself

1. For what value of a does the line $y = x + a$ touches the ellipse $9x^2 + 16y^2 = 144$.
2. Find the lengths of transverse axis and conjugate axis, eccentricity and the co-ordinates of foci and vertices; lengths of the latus rectum, equations of the directrices of the hyperbola $16x^2 - 9y^2 = -144$.
3. Find the equation of the parabola having the vertex at (0,1) and the focus at (0,0)
4. Find the equations of the tangents to the ellipse $3x^2 + 4y^2 = 12$ which are perpendicular to the line $y + 2x = 4$.
5. Determine the value of a If the straight line $x + y = 1$ is a normal to the parabola $x^2 = ay$

Answer to check your Progress

1B 2A 3C4 B 5 A 6 B 7 B 8 B 9 C 10 B 11 B 12A 13B 14 C 15 A