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RELATIONS AND FUNCTIONS-II

RELATION

Let A and B be two sets. Then a relation R from Set A into Set B is a subset of A × B.

Thus, R is a relation from A to B $\Leftrightarrow R \subseteq A \times B$

- If (a,b) $\in$ R then we write aRb which is read as 'a' is related to b by the relation R,
- If (a, b) $\notin$R then we write a~~R~~b and we say that a is not related to b bythe relation R
- If n(A) = m and n(B) = n, then A × B has mn ordered pairs, therefore, total number of relations form A to B is 2^{mn}

Types of Relations

Reflexive Relation

A relation R on a set A is said to be reflexive if every element of A is related to itself.

- Thus, R is reflexive $\Leftrightarrow$ (a, a) $\in$R for all a $\in$ A
- A relation R is not reflexive if there exists an element a$\in$ A such that (a, a) R$\notin$A

Symmetric Relation

A relation R on a set A is said to be symmetric relation if (a, b) $\in$R $\Rightarrow$(b, a) $\in$ R for all (a, b) $\in$A

i.e. aRb $\Rightarrow$ bRa for all (a, b) $\in$ A

Transitive Relation

Let A be any set. A relation R on A is said to be transitive relation if

(a, b) $\in$R and and (b, c) $\in$R$\Rightarrow$ (a, c) $\in$R for all a, b, c$\in$ A

i.e. aRb and bRc $\Rightarrow$ aRc for all a, b, c$\in$A

EQUIVALENCE RELATION

A relation R on a set A is said to be an equivalence relation on A iff

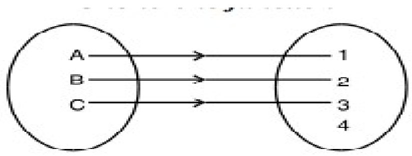
- It is reflexive i.e. (a, a) $\in$R for all a $\in$A
- It is symmetric i.e. (a, b) $\in$R $\Rightarrow$ (b, a) $\in$R for all a, b$\in$ A
- It is transitive i.e. (a, b) $\in$R and (b, c) $\in$R $\Rightarrow$ (a, c) $\in$ R for all a, b, c$\in$A

CLASSIFICATION OF FUNCTIONS

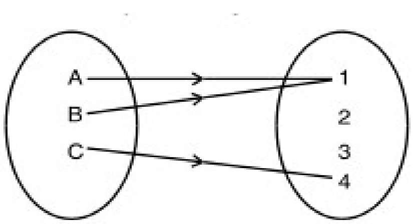
Let f be a function from A to B. If every element of the set B is the image of at least one element of the set A i.e. if there is no unpaired element in the set B then

The function f maps the set A onto the set B. Otherwise we say that the function maps the set A into the set B.

Functions for which each element of the set A is mapped to a different element of the set B are said to be **one-to-one**.



A function can map more than one element of the set A to the same element of the set B. Such a type of function is said to be **many-to-one**.

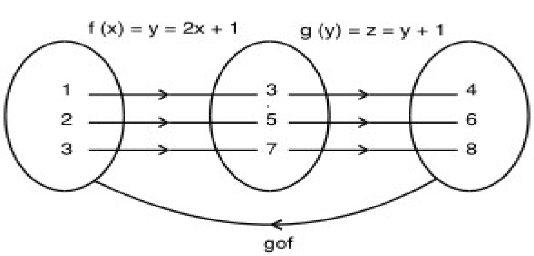


COMPOSITION OF FUNCTIONS

$y = 2x + 1$, $x \in \{ 1, 2, 3\}$

$z = y+1$, $y \in \{3, 5, 7 \}$

Then z is the composition of two functions x and y because z is defined in terms of y and y in terms of x.



The composition, say, gof of function g and f is defined as function g of function

F:A $\rightarrow$ and g : B $\rightarrow$ C

g o f : A to C

INVERSE OF A FUNCTION

- If range is a subset of co-domain that function is called on into function.
- If f: A $\rightarrow$B and f (x) = f (y) $\rightarrow$ x = y that function is called one-one function.
- Any function is invertible if it is one-one-onto or bijective
- If more than one element of A has only one image in to than function is called many one function

Binary Operation

Let A, B be two non-empty sets, then a function from A × A to A is called a binary operation on A.

If a binary operation on A is denoted by '*', the unique element of A associated with the ordered pair (a, b) of A × A is denoted by a * b.

The order of the elements is taken into consideration, i.e. the elements associated with the pairs (a, b) and (b, a) may be different i.e. a * b may not be equal to b * a.

Let A be a non-empty set and '*' be an operation on A, then

- A is said to be closed under the operation * iff for all a, b$\in$ A implies a * b $\in$A.
- The operation is said to be commutative iff a * b = b * a for all a, b $\in$A.

- The operation is said to be associative iff (a * b) * c = a * (b * c) for all a, b, c $\in$ A
- An element e $\in$A is said to be an identity element iff e * a = a = a * e
- An element a$\in$A is called invertible iff these exists some b $\in$A such that a * b = e = b * a
- a, b is called inverse of a

Check Your Progress

1. If $f : I \to I, f(x) = x^3 + 1$, then f is -
   (A) one-one but not onto
   (B) onto but not one-one
   (C) One-one onto
   (D) None of these
2. Function f : R $\to$ R, f(x) = x |x| is -
   (A) one-one but not onto
   (B) onto but not one-one
   (C) one-one onto
   (D) neither one-one nor onto
3. f : R $\to$R, $f(x) = \frac{x^2}{1+x^2}$, is -
   (A) many-one function
   (B) odd function
   (C) one-one function
   (D) None of these
4. If f : $R_0 \to R_0$, $f(x) = \frac{1}{x}$, then f is -
   (A) one-one but not onto
   (B) onto but not one-one
   (C) neither one-one nor onto
   (D) both one-one and onto
5. If f(x) = 2x and g is identity function, then-
   (A) (fog) (x) = g(x)
   (B) (g + g) (x) = g(x)
   (C) (fog) (x) = (g + g) (x)
   (D) None of these
6. gof exists, when-
   (A) domain of f = domain of g
   (B) co-domain of f = domain of g
   (C) co-domain of g = domain of
   (D) co-domain of g = co-domain of f
7. If f : R $\to$ R, $f(x) = x^2 + 2x - 3$ and g : R $\to$R, g(x) = 3x – 4, then the value of fog (x) is-
   (A) $3x^2 + 6x - 13$
   (B) $9x^2 - 18x + 5$
   (C) $(3x - 4)^2 + 2x - 3$
   (D) None of these
8. If f : R$\to$ R, $f(x) = x^2 + 3$, then pre-image of 2 under f is –
   (A) {1,–1} (B) {1}
   (B) {–1} (D) {0}
9. Which of the following functions has its inverse-
   (A) f : R $\to$R, $f(x) = a^x$
   (B) f : R$\to$R, f(x) = |x| + |x – 1|
   (C) f : $R_0 \to R^+$, f(x) = |x|
   (D) f : [π, 2π]$\to$ [–1,1], f(x) = cos x
10. If function f : R$\to R^+$, $f(x) = 2^x$, then f^{-1} (x) will be equal to-
    (A) $\log_x 2$ (B) $\log_2 (1/x)$
    (C) $\log_2 x$ (D) None of these

Stretch Yourself

1. If $f(x) = \sqrt{(2+x-x^2)}$ and $g(x) = \sqrt{-x} + \frac{1}{\sqrt{x+2}}$. Then find the domain of $f + g$.
2. Let $f : R \rightarrow R$ be a function defined by $f(x) = x + \sqrt{x^2}$, then find the nature of function f.
3. If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then find (fofof) (x)
4. Function $f : R \rightarrow {}^{R+}$, $f(x) = {}^{x2} + 2$ & $g : {}^{R+} \rightarrow R$, $g(x) = \left(1 - \frac{1}{1-x}\right)$ then find the value of gof (2).
5. If $f(x) = \log_e(x + \sqrt{1+x^2})$, Find $f^{-1}(x)$

Hint to Check Yourself

1 A	2 C	3 A	4 D	5 C
6 B	7 B	8 D	9 D	10 C