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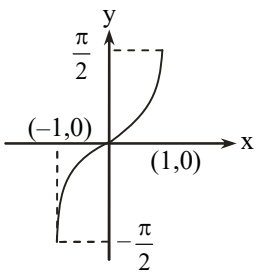
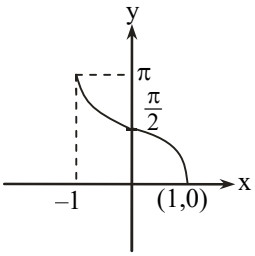
INVERSE TRIGONOMETRIC FUNCTIONS

Graph of different inverse trigonometric Function

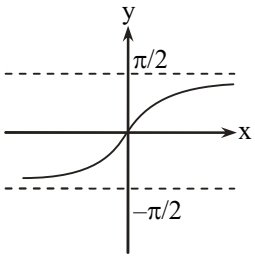
(i) $f(x) = \sin^{-1} x$



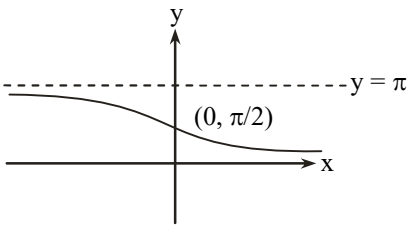
(ii) $f(x) = \cos^{-1} x$



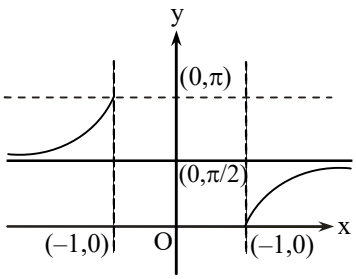
(iii) $f(x) = \tan^{-1} x$



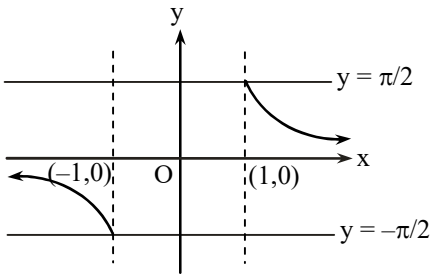
(iv) $f(x) = \cot^{-1} x$



(v) $f(x) = \sec^{-1} x$



(vi) $f(x) = \operatorname{cosec}^{-1} x$



Domain & range of Inverse Trigonometric function

Function	Domain	Range
$\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
$\operatorname{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

Properties

(i) $\sin^{-1}(\sin\theta) = \theta$,

Provided that $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

(ii) $\cos^{-1}(\cos\theta) = \theta$,

Provided that $0 \leq \theta \leq \pi$

(iii) $\tan^{-1}(\tan\theta) = \theta$,

Provided that $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(iv) $\cot^{-1}(\cot\theta) = \theta$,

Provided that $0 < \theta < \pi$

(v) $\sec^{-1}(\sec\theta) = \theta$,

Provided that $0 \leq \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \leq \pi$

(vi) $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta$

Provided that $-\frac{\pi}{2} \leq 0$ or $0 < \theta \frac{\pi}{2}$

(vii) $\csc^{-1}x = \sin^{-1}\left(\frac{1}{x}\right)$

(viii) $\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$

(ix) $\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)$

(x) $\sin^{-1}(-x) = -\sin^{-1}x$

(xi) $\cos^{-1}(-x) = -\cos^{-1}x$

(xii) $\tan^{-1}(-x) = -\tan^{-1}x$

(xiii) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

(xiv) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(xv) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

(xvi) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(xvii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(xviii) $2\tan^{-1}x = \sin^{-1}\left[\frac{2x}{1+x^2}\right] = \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right] = \tan^{-1}\left[\frac{2x}{1-x^2}\right]$

(xix) $\sin^{-1}x = \cos^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left[\frac{x}{\sqrt{1-x^2}}\right] = \sec^{-1}\left[\frac{1}{\sqrt{1-x^2}}\right] = \cot^{-1}\left[\frac{\sqrt{1-x^2}}{x}\right] = \csc^{-1}\left[\frac{1}{x}\right]$

(xx) $\cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left[\frac{\sqrt{1-x^2}}{x}\right]$

(xxi) $= \csc^{-1}\frac{1}{\sqrt{1-x^2}} = \cot^{-1}\left[\frac{x}{\sqrt{1-x^2}}\right] = \sec^{-1}\frac{1}{x}$

Check Your Progress

1. $\sin^{-1}x + \sin^{-1}\frac{1}{x} + \cos^{-1}x + \cos^{-1}\frac{1}{x} =$

(A) π (B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{2}$ (D) None of these

2. If $x > 0$, $\sin^{-1}(2\pi + x) + \cos^{-1}(2\pi + x)$

(A) $2\pi + \frac{\pi}{2}$ (B) $\frac{\pi}{2}$

(C) $x + \frac{\pi}{2}$ (D) None of these

3. $\sin^{-1}\sin 15 + \cos^{-1}\cos 20 + \tan^{-1}\tan 25 =$

(A) $19\pi - 60$ (B) $30 - 9\pi$

(C) $19 - 60\pi$ (D) $60\pi - 19$

4. $\tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc} =$

(A) $\tan^{-1}a - \tan^{-1}b$

(B) $\tan^{-1}a - \tan^{-1}c$

(C) $\tan^{-1}b - \tan^{-1}c$

(D) $\tan^{-1}c - \tan^{-1}a$

5. If $\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{2}{3} = \sin^{-1}x$, then x is equal to -

(A) 0 (B) $\frac{\sqrt{5}-4\sqrt{2}}{9}$

(C) $\frac{\sqrt{5}+4\sqrt{2}}{9}$ (D) $\frac{\pi}{2}$

6. If $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$ then x =

(A) –1 (B) $\frac{1}{6}$

(C) –1, $\frac{1}{6}$ (D) None of these

7. The value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$ is -

(A) $\frac{3\pi}{5}$ (B) $\frac{7\pi}{5}$

(C) $\frac{\pi}{10}$ (D) $-\frac{\pi}{10}$

8. If $\theta = \cot^{-1}\sqrt{\cos x} - \tan^{-1}\sqrt{\cos x}$, then $\sin\theta =$

(A) $\tan\frac{1}{2}x$ (B) $\tan^2(x/2)$

(C) $\frac{1}{2}\tan^{-1}(x/2)$ (D) None of these

9. If a, b, c be positive real numbers and the value of

$$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$$

then $\tan\theta$ is equal to -

(A) 0 (B) 1

(C) $\frac{a+b+c}{abc}$ (D) None of these

10. The value of

$\tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/2)$ is equal to -

(A) $\pi/4$ (B) $5\pi/12$

(C) $3\pi/4$ (D) $13\pi/12$

Stretch Yourself

1. Find the principal value of

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$$

2. Find the value of $\cos [\tan^{-1}\{\sin (\cot^{-1}x)\}]$
3. Find the value of

$$3 \tan^{-1}\left(\frac{1}{2}\right) + 2 \tan^{-1}\left(\frac{1}{5}\right)$$

4. If $3 \cos^{-1}(x^2 - 7x + 25/2) = \pi$, then find x
5. Find the value of
   $\cot [\tan^{-1}(1/7) + \tan^{-1}(1/13)]$

Hint to Check Yourself

1 A 2 D 3 B 4 B 5 C

6 B 7 D 8 B 8 A 10 C