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DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS

Derivative of Trigonometric Functions from First Principle

I. $\frac{d}{dx}(\sin x) = \cos x$

II. $\frac{d}{dx}(\cos x) = -\sin x$

III. $\frac{d}{dx}(\tan x) = \sec^2 x$

IV. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

V. $\frac{d}{dx}(\sec x) = \sec x \tan x$

VI. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

VII. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$, $-1 < x < 1$

VIII. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$, $-1 < x < 1$

IX. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

X. $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

XI. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$ $|x| > 1$

XII. $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

XIII. $\frac{d}{dx}(\sinh x) = \cosh x$

XIV. $\frac{d}{dx}(\cosh x) = \sinh x$

XV. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

XVI. $\frac{d}{dx}(\coth x) = -\operatorname{cosec} h^2 x$

XVII. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

XVIII. $\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosec} hx \coth x$

XIX. $\frac{d}{dx}(\sin h^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

XX. $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$, $x > 1$

XXI. $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

XXII. $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{x^2-1}$, $|x| > 1$

XXIII. $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$, $(0 < x < 1)$

XXIV. $\frac{d}{dx}(\operatorname{cosech}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$, $x \neq 0$

XXV. $\frac{d}{dx}(e^{ax} \sinh x) = e^{ax}(a \sinh x + b \cos bx)$

Check Yourself

1. If $y = \tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$, then $\frac{dy}{dx}$ equals-

(A) $\frac{1}{1+x^2}$ (B) $\frac{2}{1+x^2}$

(C) $\frac{1}{2(1+x^2)}$ (D) $-\frac{1}{2(1+x^2)}$

2. If $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right) + \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$, then dy/dx equals -

(A) 0 (B) $\frac{1}{\sqrt{1-x^2}}$

(C) $\frac{2}{\sqrt{1-x^2}}$ (D) $\frac{3}{\sqrt{1-x^2}}$

3. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)\right]$ equal

(A) 1/2 (B) –1

(C) 1/4 (D) – 1/2

4. $\frac{d}{dx}[\cos^{-1}\sqrt{x} + \sin^{-1}\sqrt{1-x}]$ equals-

(A) 1 (B) 0

(C) $-\frac{1}{\sqrt{1-x}}$ (D) $-\frac{1}{\sqrt{x-x^2}}$

5. If $y = \sec\sqrt{a+bx}$, then $\frac{dy}{dx}$ equals-

(A) $\frac{b}{b\sqrt{a+bx}}\sec\sqrt{a+bx}\tan\sqrt{a+bx}$

(B) $\frac{b}{2\sqrt{a+bx}}\sec\sqrt{a+bx}\tan\sqrt{a+bx}$

(C) $2b\sqrt{a+bx}\sec\sqrt{a+bx}\tan\sqrt{a+bx}$

(D) None of these

6. If $y = \sqrt{\frac{1+\tan x}{1-\tan x}}$, then $\frac{dy}{dx}$ is equal to-

(A) $\frac{1}{2}\sqrt{\frac{1-\tan x}{1+\tan x}}\sec^2\left(\frac{\pi}{4}+x\right)$

(B) $\sqrt{\frac{1-\tan x}{1+\tan x}}\sec^2\left(\frac{\pi}{4}+x\right)$

(C) $\frac{1}{2}\sqrt{\frac{1-\tan x}{1+\tan x}}\sec\left(\frac{\pi}{4}+x\right)$

(D) None of these

7. If $y = \sin x + \cos 2x$, then $\frac{d^2y}{dx^2}$ equals-

(A) sin x + 4 sin 2x

(B) – (sin x + 4 cos 2x)

(C) – sin x + 4 cos 2x

(D) None of these

8. $\frac{d}{d\theta}\left\{\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)\right\}$ equals-

(A) 1/2 (B) 1

(C) sec θ (D) cosec θ

9. Let $y = \cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$, $\frac{\pi}{2} < x < \pi$. Then $\frac{dy}{dx}$

(A) 1/2 (B) – 1/2

(C) Does not exist (D) None of these

10. If sin y = x cos (a + y), then dy/dx equals-

(A) $\frac{\cos^2(a+y)}{\cos a}$ (B) $\frac{\cos a}{\cos^2(a+y)}$

(C) $\frac{\cos(a+y)}{\cos^2 a}$ (D) None of these

Stretch Yourself

1. If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $z = \tan^{-1} x$, then find $\frac{dy}{dz}$

2. If $y = \tan^{-1}\left(\frac{4x}{1-4x^2}\right)$, Find $\frac{dy}{dx}$

3. Find $\frac{d}{dx}\left[\sin^{-1}\left(\frac{3x}{2}-\frac{x^3}{2}\right)\right]$

4. Find $\frac{d}{dx}\cot^{-1}\left(\frac{1+x}{1-x}\right)$

5. Find $\frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{a-x}{a+x}}\right]$

6. If $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right) + \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$, then find dy/dx

Hint to check yourself

1 C 2 D 3 A 4 D 5 B

6 A 7 B 8 A 9 B 10 A