

DIFFERENTIATION OF EXPONENTIAL AND LOGARITHMIC

DERIVATIVE OF EXPONENTIAL FUNCTIONS

- I. $\frac{d}{dx} e^x = e^x$
- II. $\frac{d}{dx} e^{(ax+b)} = ae^{(ax+b)}$
- III. $\frac{d}{dx} (e^{ax} \sin b x)$
 $= e^{ax} (a \sin b x + b \cos b x)$
 $= \sqrt{a^2 + b^2} e^{ax} \sin (bx + \tan^{-1} b/a)$
- IV. $\frac{d}{dx} (e^{ax} \cos b x)$
 $= e^{ax} (a \cos b x - b \sin b x)$
 $= \sqrt{a^2 + b^2} e^{ax} \cos (bx + \tan^{-1} b/a)$

DERIVATIVE OF LOGARITHMIC FUNCTIONS

Let $y = [f(x)]^{g(x)}$
 $\log y = g(x) \log [f(x)]$
 $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx} g(x) \cdot \log [f(x)]$
 $\frac{dy}{dx} = [f(x)]^{g(x)} \cdot \left\{ \frac{d}{dx} [g(x) \log f(x)] \right\}$

- I. $\frac{d}{dx} (\log_e x) = 1/x$
- II. $\frac{d}{dx} (\log_a x) = \frac{1}{x \log a}$
- III. $\frac{d}{dx} (\log_e x) = 1/x$

Check Your Progress

1. If $y = \log_x 10$, then the value of dy/dx equals-
- (A) $1/x$ (B) $10/x$
 (C) $-\frac{(\log_x 10)^2}{x \log_e 10}$ (D) $\frac{1}{(x \log_e 10)}$

2. If $(a + bx)e^{y/x} = x$, then the value of $x^3 \frac{d^2y}{dx^2}$ is
- (A) $\left(y \frac{dy}{dx} - x \right)^2$

(B) $\left(x \frac{dy}{dx} - y \right)^2$

(C) $x \frac{dy}{dx} - y$

(D) None of these

3. $\frac{d}{dx} \left[\log \left\{ e^x \left(\frac{x-2}{x+2} \right)^{3/4} \right\} \right]$ equals -

(A) $\frac{x^2 - 1}{x^2 - 4}$ (B) 1

(C) $\frac{x^2 + 1}{x^2 - 4}$ (D) $e^x \frac{x^2 - 1}{x^2 - 4}$

4. If $f(x) = \log_x (\square n x)$, then at $x = e$, $f'(x)$ equals-

- (A) 0 (B) 1
(C) e (D) 1/e

5. If $f(x) = \log_a (\log_a x)$, then $f'(x)$ is equal to -

- (A) $\frac{\log a}{x}$ (B) $\frac{\log a}{x \log x}$
(C) $\frac{\log_a e}{x \log_e x}$ (D) $\frac{x}{\log a}$

6. If $y = \log \sqrt{\left(\frac{1+\cos x}{1-\cos x}\right)}$, then the value of $\frac{dy}{dx}$ is-

- (A) $\sec x$ (B) $\operatorname{cosec} x$
(C) $-\sec x$ (D) $-\operatorname{cosec} x$

7. If $y = \log (\operatorname{cosec} x - \cot x)$, then $\frac{dy}{dx}$ equals-

- (A) $\operatorname{cosec} x + \cot x$
(B) $\cot x$
(C) $\sec x + \tan x$
(D) $\operatorname{cosec} x$

8. If $y = \log \sqrt{\frac{1-\sin x}{1+\sin x}}$, then $\frac{dy}{dx}$ equals -

- (A) $\sec x$ (B) $-\sec x$
(C) $\operatorname{cosec} x$ (D) $\sec x \tan x$

9. If $y = e^{3x} \sin 4x$, then the value of $\frac{dy}{dx}$ is-

- (A) $e^{3x} \sin \left(4x + \tan^{-1} \frac{4}{3}\right)$
(B) $e^{3x} \cos \left(4x + \tan^{-1} \frac{4}{3}\right)$
(C) $5e^{3x} \sin \left(4x + \tan^{-1} \frac{4}{3}\right)$

(D) $5e^{3x} \cos \left(4x + \tan^{-1} \frac{4}{3}\right)$

10. If $y = \log_{10} (\sin x)$, then $\frac{dy}{dx}$ equals-

- (A) $\sin x \log_{10} e$
(B) $\cos x \log_{10} e$
(C) $\cot x \log_{10} e$
(D) $\cot x$

11. $d/dx \log \left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)$ equals -

- (A) $\frac{\sqrt{x}}{1-x}$ (B) $\frac{1}{1-x}$

- (C) $\frac{1}{\sqrt{x}(1-x)}$ (D) None of these
12. $d/dx [e^x \sin \sqrt{3} x]$ equals-
- (A) $e^x \sin (\sqrt{3} x + \pi/3)$
 (B) $2e^x \sin (\sqrt{3} x + \pi/3)$
 (C) $\frac{1}{2} e^x \sin (\sqrt{3} x + \pi/3)$
 (D) $\frac{1}{2} e^x \sin (\sqrt{3} x - \pi/3)$
13. If $y = x^3 \log x$, then the value of $\frac{d^2y}{dx^2}$ is-
- (A) $5x + 6x \log x$
 (B) $3x + 6x \log x$
 (C) $6x + 5x \log x$
 (D) None of these
14. If $y = \log(1-t^2)$ and $x = \sin^{-1}(t)$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{1}{2}$ is-
- (A) $3/8$ (B) $-3/8$
 (C) $8/3$ (D) None of these
15. $d/dx (\log_{10}x)$ equals –
- (A) $1/x$ (B) $(1/x) \log_{10}e$
- (C) $\log \log_e x$ (D) None of these
16. The derivative of $\log_a x + \log x^a$ is equal to–
- (A) $\frac{1}{x} (a + \log_a e)$
 (B) $\frac{1}{x} (a + \log_e a)$
 (C) $\frac{a}{x} + \log_a e$
 (D) None of these
17. If $y \sin x = x + y$, then $\left(\frac{dy}{dx}\right)_{x=0}$ equals-
- (A) 1 (B) -1
 (C) 0 (D) 2
18. If $x^3 + y^3 = 3xy$, then the value of $\frac{dy}{dx}$ is-
- (A) $\frac{x-y^2}{x^2-y}$ (B) $\frac{x^2-y}{x-y^2}$
 (C) $\frac{x-y^2}{y-x^2}$ (D) $\frac{x^2-y}{y^2-x}$
19. If $ax^2 + 2hxy + by^2 = 0$, then $\frac{dy}{dx}$ equals-
- (A) $\frac{ax+hy}{hx+by}$ (B) $-\frac{ax+hy}{hx+by}$

(C) $\frac{hx+by}{ax+hy}$ (D) $-\frac{hx+by}{ax+hy}$

20. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ equals-

(A) e^{x-y} (B) e^{y-x}
(C) $-e^{x-y}$ (D) $-e^{y-x}$

Hint to check your progress

- 1 C 2 B 3 A 4 D 5 C
6 D 7 D 8 B 9 C 10 C
11 C 12 B 13 A 14 D 15 B
16 A 17 B 18 B 19 B 20 D

Stretch Yourself

1. If $y^2 x + x^2 y + 3xy = 2$, find $\frac{dy}{dx}$
2. If $e^x \sin y - e^y \cos x = 1$, find $\frac{dy}{dx}$
3. What is the derivative of x^{a^x}
4. If $y = \log(x^x)$, find $\frac{dy}{dx}$
5. Find $d/dx (x^{\log x})$
6. If $y = e^{ax+b}$, Find $(y_2)_0$

7. If $y = x^x$, then Find the value of $\frac{dy}{dx}$
8. If $y = e^{\log(\sin^{-1} x)}$, then Find $\frac{dy}{dx}$
9. If $y = \left(1 + \frac{1}{x}\right)^x$, then find $\frac{dy}{dx}$
10. If $y = x^x + a^x + x^a$, then find $\frac{dy}{dx}$ at $x = 1$