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DEFINITE INTEGRALS

If $\frac{d}{dx}[f(x)] = \phi(x)$ and a and b, are two values independent of variable x, then

$$\int_a^b \phi(x)\,dx = [f(x)]_a^b = f(b) - f(a)$$

is called **Definite Integral** of ϕ (x) within limits a and b. Here **a** is called the **lower limit** and **b** is called the **upper limit** of the integral. The interval [a, b] is known as **range of integration**. It should be noted that every definite integral has a unique value.

Properties

[P-1] $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

i.e. the value of a definite integral remains
unchanged if its variable is placed by any other symbol.

[P-2] $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

i.e. the interchange of limits of a definite integral changes only its sign.

[P-3] $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

where a < c < b.

or $\int_a^b f(x)\,dx = \int_a^{c_1} f(x)\,dx + \int_{c_1}^{c_2} f(x)\,dx +$ ....+ $\int_{c_n}^b f(x)\,dx$ where $a < c_1 < c_2 < ....c_n < b$.

[P-4] $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

[P-5] $\int_{-a}^a f(x)\,dx$

$$= \begin{cases} 0, \text{ if } f(-x) = -f(x) \text{ i.e. if } f(x) \text{ is odd} \\ 2\int_0^a f(x)\,dx, \text{ if } f(-x) = f(x) \text{ i.e. if } f(x) \text{ even} \end{cases}$$

This property is generally used when integrand is either even or odd function of x.

[P-6]

$$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, \text{ if } f(2a-x) = f(x) \\ 0, \quad \text{if } f(2a-x) = -f(x) \end{cases}$$

It is generally used to make half the upper limit.

[P-7] If f(x) = f(x + a) , then

$$\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx$$

[P-8] $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

[P-9] $\frac{d}{dt}\left[\int_{\phi(t)}^{\psi(t)} f(x)\,dx\right] = f\{\psi(t)\}\psi'(t) - f\{\phi(t)\}\phi'(t)$

Check Your Progress

1. $\int_0^{\pi/4} \tan^2 x\,dx$ equals-

(A) $\pi/4$ (B) $1 + (\pi/4)$

(C) $1 - (\pi/4)$ (D) $1 - (\pi/2)$

2. The value of $\int_0^{2a} \frac{dx}{\sqrt{2ax - x^2}}$ is-

(A) π (B) $\pi/2$

(C) $\pi/4$ (D) 2π

3. The value of $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2}\,dx$ is-

(A) log (9/8) (B) log (4/3)

(C) log (3/4) (D) None of these

4. $\int_0^{\infty} \frac{e^{\tan^{-1} x}}{1+x^2}\,dx$ equals-

(A) 1 (B) $e^{\pi/2} + 1$

(C) $e^{\pi/2} - 1$ (D) None of these

5. $\int_0^{\pi/4} \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}\,dx$ equals -

(A) $\log_e \frac{2}{3}$ (B) $\log_e 3$

(C) $\frac{1}{2}\log_e \frac{4}{3}$ (D) $\log_e \frac{4}{3}$

6. $\int_0^{\pi/4} \tan^4 x\,dx$ equals -

(A) $\frac{\pi}{4} + \frac{2}{3}$ (B) $\frac{\pi}{4} - \frac{2}{3}$

(C) $\frac{\pi}{4} + \frac{1}{3}$ (D) $\frac{\pi}{4} - \frac{1}{3}$

7. $\int_1^3 \left(\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}\right)dx$, equals-

(A) π (B) 2π

(C) 3π (D) None of these

8. $\int_{-\pi/4}^{\pi/2} e^{-x} \sin x\,dx =$

(A) $-\frac{1}{2} e^{-\pi/2}$ (B) $-\frac{\sqrt{2}}{2} e^{-\pi/4}$

(C) $-\sqrt{2}\,(e^{-\pi/4} + e^{-\pi/2})$ (D) 0

9. $\int_0^{\pi/4} \frac{\sqrt{\tan x}}{\sin x \cos x}\, dx$ equals-

(A) 1 (B) 2

(C) 0 (D) 4

10. $\int_{\pi/6}^{\pi/4} \frac{\tan x + \cot x}{\tan^{-1} x + \cot^{-1} x}\, dx$ equals-

(A) 0 (B) $(\sqrt{3}+1)/\sqrt{3}$

(C) $(\log 3)/\pi$ (D) None of these

11. $\int_0^3 \sqrt{\frac{x^3}{3-x}}\, dx$ equals-

(A) $3\pi/16$ (B) $27\pi/8$

(C) $3\pi/32$ (D) $9\pi/8$

12. If $f(x) = \begin{cases} x^2, & \text{when } 0 \le x < 1 \\ \sqrt{x}, & \text{when } 1 \le x < 2 \end{cases}$, then $\int_0^2 f(x)\, dx$ equals-

(A) $\frac{1}{3}(4\sqrt{2} - 1)$

(B) $\frac{1}{3}(4\sqrt{2} + 1)$

(C) 0

(D) does not exist

13. $\int_0^1 |3x - 1|\, dx$ equals-

(A) 5/6 (B) 5/3

(C) 10/3 (D) 5

14. $\int_0^{\pi} |\cos x|\, dx$ equals -

(A) 1 (B) 2

(C) 0 (D) –1

15. $\int_{1/e}^{e} |\log x|\, dx =$

(A) $e^{-1} - 1$ (B) 2 (1–1/e)

(C) 1 – 1/e (D) None of these

16. $\int_{-\pi/2}^{\pi/2} (\sin^3 x + \cos^3 x)\, dx$ equals-

(A) 0 (B) 1/3

(C) 4/3 (D) 2/3

17. $\int_{-\pi/2}^{\pi/2} \frac{dx}{1+\cos x}$ equals-

(A) 0 (B) 2

(C) 1 (D) 3

18. $\int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin\theta}{2+\sin\theta}\right) d\theta$ equals-

(A) 0 (B) 1

(C) 2 (D) None of these

19. $\int_{0}^{2\pi} \frac{\sin 2\theta}{a-b\cos\theta} d\theta$ equals-

(A) 1 (B) 2

(C) π/4 (D) 0

20. $\int_{0}^{400\pi} \sqrt{1-\cos 2x}$ dx is equal to-

(A) 400 $\sqrt{2}$ (B) 800 $\sqrt{2}$

(C) 0 (D) None of these

21. Which of the following is correct?

(A) $\int_0^a f(x)dx = \int_0^a f(a+x)dx$

(B) $\int_0^{2a} f(x)dx = \int_0^a f(x)dx$

(C) $\int_0^a f(x)dx = \int_0^{-a} f(-x)dx$

(D) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

22. $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} dx$ is equal to-

(A) 2/1 (B) 3/4

(C) 1/2 (D) None of these

23. $\int_0^{\pi/2} \frac{\phi(x)}{\phi(x)+\phi(\pi/2-x)} dx$ is equal to-

(A) π4 (B) π/2

(C) π (D) None of these

24. $\int_0^{\pi/2} \log \cos x \, dx$ equals-

(A) (π/2) log (1/2)

(B) πlog 2

(C) –πlog 2

(D) 2πlog 2

25. $\int_0^{\pi/2} \sin^2\theta \cos^6\theta \, d\theta$ equals-

(A) –π/16 (B) π/16

(C) $5\pi/256$ (D) $-5\pi/256$

$\int_0^{\pi/2} \sin^5 x\,dx$ equals-

1C 2A 3 A 4 C 5 D

6 B 7A 8 A 9 B 10 C

11 B 12A 13 A 14 B 15 B

16 C 17 B 18 A 19 D 20 B

21D 22C 23A 24 A 25 C

Stretch Yourself

Find

1. $\int_0^{2\pi} \sin^4 x \cos^6 x\,dx$
2. If $\int_0^{\pi} \log \sin x\,dx = k$, then the value of k
3. If $f(x) = |x| + |x - 1|$, then $\int_0^2 f(x)dx$
4. $\int_0^{\pi/4} \cos^{3/2} 2\pi \cos \pi\, d\pi$
5. $\int_0^1 e^{x^2} (x - \pi)dx = 0$, then x
6. $\int_0^{\pi} \log (1 - \cos x)\,dx$
7. $\int_1^5 (|x-3| + |1-x|)\,dx$
8. $\int_0^a x^4 \sqrt{a^2 - x^2}\,dx$

Hint to Check Your Progress