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Differential Equation

An equation containing an independent variable, dependent variable and differential coefficients of dependent variable with respect to independent variable is called a **differential equation.**

For Example-

(i) $\frac{dy}{dx} = \sin x$

(ii) $\frac{dy}{dx} + xy = \cot x$

(iii) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$

(iv) $\left(\frac{d^2y}{dx^2}\right)^2 + x^2\left(\frac{dy}{dx}\right)^3 = 0$

(v) $\frac{d^2y}{dx^2} + \left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = 0$

Order of differential equation

The order of a differential equation is the order of the highest derivative occurring in the differential equation.

Degree of differential equation

The degree of a differential equation is the degree of the highest order derivative when differential coefficients are free from radical and fraction

The general and n^{th} order differential equation is given below -

$$P_0\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + \ldots + P_{n-1}\frac{dy}{dx} + P_n y = Q$$

where $P_0, P_1, P_2, \ldots\ldots, P_{n-1}$ and Q are either constants or functions of independent variable x.

Those equations which are not linear are called **non- linear differential equations.**

FORMATION OF A DIFFERENTIAL EQUATION

(i) Write down the given equation.

(ii) Differentiate it successively with respect to x that number of times equal to the arbitrary constants.

(iii) Hence on eliminating arbitrary constants results a differential equation which involves

$$x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2} \ldots\ldots$$

Differential equations of the form $\frac{dy}{dx} = f(x)$.

$$\frac{dy}{dx} = f(x) \Rightarrow dy = f(x)\,dx.$$

Integrating both sides we obtain

$$\int dy = \int f(x)\,dx + c$$

$$\text{or } y = \int f(x)\,dx + c$$

Differential equations of the form dy/dx=f(x) g(y)

$$\frac{dy}{dx} = f(x)\,g(y)$$

$$\int \frac{dy}{g(y)} = \int f(x)\,dx + c.$$

Differential Equation of homogeneous type

An equation in x and y is said to be homogeneous if it can be put in the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ where f(x, y) and g(x, y) are both homogeneous functions of the same degree in x & y.

So to solve the homogeneous differential equation $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$,substitute y = vx and

so $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Thus $v + x\frac{dv}{dx} = f(v) \Rightarrow \frac{dx}{x} = \frac{dv}{f(v) - v}$

Therefore solution is $\int \frac{dx}{x} = \int \frac{dv}{f(v) - v} + c$

Check Your Progress

1. A differential equation of first order and first degree is-

(A) $x\left(\frac{dy}{dx}\right)^2 - x + a = 0$

(B) (B) $\frac{d^2y}{dx^2} + xy = 0$

(C) dy + dx = 0

(D) None of these

2. The order and degree of differential equation $\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$ are respectively-

(A) 1, 2 (B) 1, 1

(C) 2, 1 (D) 2, 2

3. Which of the following equation is linear?

(A) $\frac{dy}{dx} + xy^2 = 1$

(B) $x^2\frac{dy}{dx} + y = e^x$

(C) $\frac{dy}{dx} + 3y = xy^2$

(D) $x\frac{dy}{dx} + y^2 = \sin x$

4. Which of the following equation is non- linear-

(A) $\frac{dy}{dx} = \cos x$

(B) $\frac{d^2y}{dx^2} + y = 0$

(C) $dx + dy = 0$

(D) $x\frac{dy}{dx} + \frac{3}{dy/dx} = y^2$

5. $y = 4 \sin 3x$ is a solution of the differential equation-

(A) $\frac{dy}{dx} + 8y = 0$

(B) $\frac{dy}{dx} - 8y = 0$

(C) $\frac{d^2y}{dx^2} + 9y = 0$

(D) $\frac{d^2y}{dx^2} - 9y = 0$

6. The differential equation of the family of curves represented by the equation $x^2 + y^2 = a^2$ is-

(A) $x + y\frac{dy}{dx} = 0$

(B) $y\frac{dy}{dx} = x$

(C) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

(D) None of these

7. The general solution of the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$ is-

(A) $x^3 - y^3 = c$

(B) $x^3 + y^3 = c$

(C) $x^2 + y^2 = c$

(D) $x^2 - y^2 = c$

8. The general solution of the equation $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$ is-

(A) $(e^y + 1)\cos x = c$

(B) $(e^y - 1)\sin x = c$

(C) $(e^y + 1)\sin x = c$

(D) None of these

9. The solution of the differential equation $dy = \sec^2 x\, dx$ is-

(A) $y = \sec x \tan x + c$

(B) $y = 2 \sec x + c$

(C) $y = \frac{1}{2}\tan x + c$

(D) None of these

10. The solution of the equation $\frac{dy}{dx} = (x + y)^2$ is-

(A) $x + y + \tan(x + c) = 0$

(B) $x - y + \tan(x + c) = 0$

(C) $x + y - \tan(x + c) = 0$

(D) None of these

11. The solution of the differential equation $\frac{dy}{dx} = \cot^2(x + y)$ is-

(A) $y = x + 1/2 \sin 2(x + y) + c$

(B) $y = x - 1/2 \sin 2(x + y) + c$

(C) $y = x + 1/2 \cos 2(x + y) - c$

(D) None of these

12. The solution of the differential equation, $\frac{dy}{dx} + \frac{y}{x} = x^2$ is-

(A) $4xy = x^4 + c$

(B) $xy = x^4 + c$

(C) $\frac{1}{4}xy = x^4 + c$

(D) $xy = 4x^4 + c$

13. The solution of the differential equation $\frac{dy}{dx} + y = \cos x$ is-

(A) $y = \frac{1}{2}(\cos x + \sin x) + ce^{-x}$

(B) $y = \frac{1}{2}(\cos x - \sin x) + ce^{-x}$

(C) $y = \cos x + \sin x + ce^{-x}$

(D) None of these

14. The integrating factor of the differential equation $(x \log x)\frac{dy}{dx} + y = 2 \log x$ is-

(A) $\log x$ (B) $\log (\log x)$

(C) e^x (D) x

15. The equation of the curve passing through the origin and satisfying the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$ is-

(A) $(1 + x^2)y = x^3$

(B) $2(1 + x^2)y = 3x^3$

(C) $3(1 + x^2)y = 4x^3$

(D) None of these

Stretch Yourself

Find

1. The solution of the equation

$$(1 - x^2)\,dy + xy\,dx = xy^2\,dx$$

2. The solution of

$$\frac{dy}{dx} = \frac{e^x(\sin^2 x + \sin 2x)}{y(2 \log y + 1)}$$

3. The solution of

$$(x\sqrt{1+y^2})dx + (y\sqrt{1+x^2})dy = 0$$

4. The solution of the differential equation

5. $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

6. The solution of $ydx - x\,dy + 3x^2 e^{x^3} y^2 dx = 0$

7. The solution of the differential equation

$$xdy - ydx = \sqrt{x^2 + y^2}\,dx$$

Hint to Check Your Progress

1C 2B 3B 4D 5C

6A 7A 8C 9D 10C

11 A 12A 13 A 14A 15C