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VECTOR

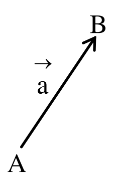
- A physical quantity which is completely specified by its magnitude only is called **scalar**. It is represented by a real number along with suitable unit.

  For example, Distance, Mass, Length, Time, Volume, Speed, Area are scalars.
- A physical quantity which has magnitude as well as direction is called a **vector.** For example, Displacement, velocity, acceleration, force etc. are vector quantities.

Representation of vector

Geometrically a vector is represented by $\vec{a}$ directed line segment. If for a vector $\vec{a}$, $\vec{a} = \overrightarrow{AB}$, then **A** is called its **initial point** and **B** is called its **terminal point**. Clearly $\overrightarrow{AB}$ and $\overrightarrow{BA}$ represents different line segments

If $\vec{a} = \overrightarrow{AB}$, then its magnitude is expressed by $|\vec{a}|$ or $|\overrightarrow{AB}|$ or AB.



Types of Vector

(i) Zero or null Vector :

A vector whose magnitude is zero is called **zero or null vector** and it is denoted by **0** or $\vec{0}$. The initial and terminal points of the directed line segment representing zero vector are coincident and its direction is arbitrary.

(ii) Unit vector :

A vector of unit magnitude is called a **unit vector**. A unit vector in the direction of a is denoted by $\hat{a}$. Thus

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{a} = \frac{vector\, a}{magnitude\, of\ a}$$

(i) $|\hat{a}| = 1$

(ii) Unit vectors parallel to x-axis, y-axis and
z-axis are denoted by i, j and k respectively.

(iii)Two unit vectors may not be equal unless they have the same direction.

(iii)Equal Vector :

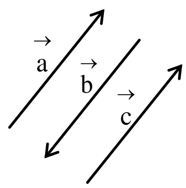
Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal , if

(a) $|\vec{a}| = |\vec{b}|$

(b) they have the same direction

(iv) Collinear vectors or Parallel Vectors :

Vectors which are parallel to the same line are called collinear vectors or parallel vectors. Such vectors have either same direction or opposite direction. If they have the same direction they are said to be like vectors, and if they have opposite directions, they are called unlike vectors.



In the diagram $\vec{a}$ and $\vec{c}$ are like vectors whereas $\vec{a}$ and $\vec{b}$ are opposite vectors.

(v) Coplanar Vectors :

If the directed line segment of some given vectors lie in a plane then they are called **coplanar vectors**.

(vi) Position Vectors :

The vector $\overrightarrow{OA}$ which represents the position of the point A with respect to a fixed point (called origin) O is called position vector of the point A. If (x,y,z) are coordinates of the point A, then

$$\overrightarrow{OA} = x\hat{i} + y\hat{j} + z\hat{k}$$

(vii) Reciprocal vectors :

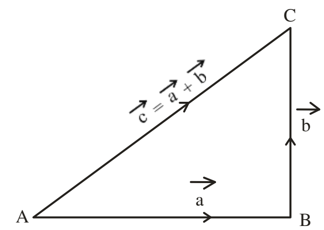
A vector which has the same direction as vector a but whose magnitude is the reciprocal of the magnitude of a, is called the reciprocal vector of vector a and is denoted by a^{-1}.

Thus if $a = \square\,\hat{a}$, then

$$a^{-1} = \frac{1}{\alpha}.\,\hat{a} = \frac{\alpha\hat{a}}{|\vec{a}|^2} = \frac{\vec{a}}{|\vec{a}|^2}$$

Triangle law of addition :

If two vectors are represented by two consecutive sides of a triangle then their sum is represented by the third side of the triangle but in opposite direction. This is known as the **triangle law of addition of vectors**.

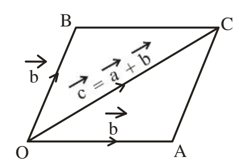


Thus, if $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{BC} = \vec{b}$, and $\overrightarrow{AC} = \vec{c}$

then $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ i.e. $\vec{a} + \vec{b} = \vec{c}$

Parallelogram Law of Addition :

If two vectors are represented by two adjacent sides of a parallelogram, then their sum is represented by the diagonal of the parallelogram whose initial point is the same as the initial point of the given vectors. This is known as **parallelogram law of addition of vectors.**



Thus if $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, and $\overrightarrow{OC} = \vec{c}$

then $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$ i.e. $\vec{a} + \vec{b} = \vec{c}$

Where $\overrightarrow{OC}$ is a diagonal of the parallelogram OABC.

Addition in component form :

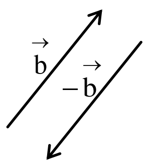
If the vectors are defined in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$. i.e. if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then their sum is defined as

$\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$

Subtraction of Vector

If $\vec{a}$ and $\vec{b}$ are two vectors, then their subtraction $\vec{a} - \vec{b}$ is defined as $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$



If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then $\vec{a} - \vec{b} = (\vec{a}_1 - \vec{b}_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 + b_3)\hat{k}$

Position Vector

If $\overrightarrow{AB}$ be any given vector and also suppose that the position vectors of initial point A and terminal point B are a and b respectively,

then $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a}$

Multiplication of Vector by scalar

If $\vec{a}$ is a vector and m is a scalar (i.e. a real number) then ma is a vector

If $\vec{a}, \vec{b}$ are any two vectors and m, n are any scalar then

(i) $m(\vec{a}) = (\vec{a})\, m = m\,\vec{a}$ (commutativity)

(ii) $m(n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$ (Associativity)

$$\left.\begin{aligned}&\text{(iii) } (m+n)\vec{a} = m\vec{a} + n\vec{a}\\ &\text{(iv) } m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}\end{aligned}\right\}\text{(Distributivity)}$$

Collinearity of vector

If $\vec{a}, \vec{b}, \vec{c}$ be position vectors of three points A,B and C respectively and x, y,z be three scalars so that all are not zero, then the necessary and sufficient conditions for three points to be collinear is that

$x\vec{a} + y\vec{b} + z\vec{c} = 0$ and $x + y + z = 0$

Coplanar vector

If $\vec{a}, \vec{b}, \vec{c}$ be three coplanar vectors, then a vector c can be expressed uniquely as linear combination of remaining two vectors i.e.

$\vec{c} = \lambda\vec{a} + \mu\,\vec{b}$

Where λ and μ are suitable scalars.

Direction cosine

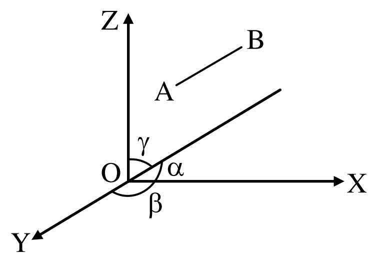
Direction cosines of a vector are the cosines of the angles subtended by the vector with the positive directions of x, y and z axes respectively.

Direction cosines of a line [Dc's] :

The cosines of the angles made by a line with coordinate axes are called the direction cosines of that line.

Let α, β, γ be the angles made by a line AB with coordinate axes then

$\cos\alpha$,, $\cos\beta$, $\cos\gamma$ are the direction cosines of AB which are generally denoted by ℓ, m, n. Hence $\ell = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$



Direction cosines of coordinate axes :

x-axis makes 0°, 90° and 90° angles with three coordinate axes, so its direction cosines are cos 0°, cos 90°, cos 90°, i.e. 1, 0, 0.

Similarly direction cosines of y-axis and z-axis are 0, 1, 0 and 0, 0, 1 respectively. Hence

dc's of x - axis = 1, 0, 0

dc's of y - axis = 0, 1, 0

dc's of z - axis = 0, 0, 1

(i) The direction cosines of a line parallel to any coordinates axis are equal to the direction cosines of the corresponding axis.

(ii) Relation between dc's : $l^2 + m^2 + n^2 = 1$

Direction ratios of a line [DR's]

Three numbers which are proportional to the direction cosines of a line are called the direction ratios of that line. If a, b, c are such numbers which are proportional to the direction cosines ℓ, m, n of a line then a, b, c are direction ratios of the line. Hence

a, b, c dr's $\frac{a}{\ell} = \frac{b}{m} = \frac{c}{n}$

Further we may observe that in above case

$$\frac{\ell}{a} = \frac{m}{b} = \frac{n}{c} \pm \frac{\sqrt{\ell^2 + m^2 + n^2}}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$\ell = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}},$$

$$m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}},$$

$$n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

.

Direction cosines of a line joining two points:

Let P (x_1, y_1, z_1) and Q (x_2, y_2, z_2); then

(i) dr's of PQ : $(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$

(ii) dc's of PQ : $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$

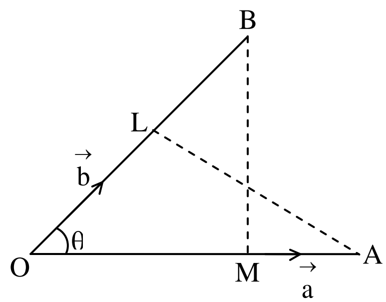
i.e.

$$\frac{x_2 - x_1}{\sqrt{\Sigma(x_2 - x_1)^2}}, \frac{y_2 - y_1}{\sqrt{\Sigma(x_2 - x_1)^2}}, \frac{z_2 - z_1}{\sqrt{\Sigma(x_2 - x_1)^2}}$$

Scalar product

If $\vec{a}$ and $\vec{b}$ are two non zero vectors and θ be the angle between them, then their **scalar product** (or dot product) is defined as the number a b $\cos\theta$ where a and b are modulii of $\vec{a}$ and $\vec{b}$ respectively .It is denoted by $\vec{a}\cdot\vec{b}$. Thus

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = a\,b\ \cos\theta$$



(i) If $\vec{a}$ and $\vec{b}$ are like vectors, then $\theta = 0$ so
$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}| = a\ b$ i.e. scalar product of two like vectors is equal to the product of their modulii

(ii) If $\vec{a}$ and $\vec{b}$ are unlike vectors then $\theta = \pi$ so $\vec{a}\cdot\vec{b} = a\ b\ \cos\pi = -\ a\ b$.

(iii) The scalar product of a vector by itself is equal to the square of its modulus i.e. $\vec{a}\,.\,\vec{a} = |\vec{a}|^2$

(iv) If $\vec{a}$ and $\vec{b}$ are perpendicular to each other then $\theta = \pi/2$, so

$$\vec{a}\cdot\vec{b} = a\ b\ \cos\pi/2 = 0$$

i.e. the scalar product of two perpendicular vectors is always zero.

But its converse may not be true i.e.

$$\vec{a}\cdot\vec{b} = 0 \not\Rightarrow \vec{a}\ \square\ \vec{b}$$

But if a and b are non zero vectors, then

$$\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{a}\perp\vec{b}$$

(v) With the help of the above cases, we get the following important results:

(a) $\hat{i}\,.\,\hat{i} = \hat{j}\,.\,\hat{j} = \hat{k}\,.\,\hat{k} = 1$ (b) $\hat{i}\,.\,\hat{j} = \hat{j}\,.\,\hat{k}\,.\,\hat{k}\,.\,\hat{j} = 0$

(vi) If $\vec{a}$ and $\vec{b}$ are unit vectors, then

$$\vec{a}\cdot\vec{b} = \cos\theta$$

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are any vectors and m, n any scalars then

(i) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (Commutativity)

(ii) $(m\vec{a}).\vec{b} = \vec{a}\,.\,(m\vec{b}) = m\,(\vec{a}\,.\,\vec{b})$

(iii) $(m\vec{a}).\,(n\vec{b}) = (mn)\,(\vec{a}\,.\,\vec{b})$

(iv) $\vec{a}\,.(\vec{b} + \vec{c}) = \vec{a}\,.\,\vec{b} + \vec{a}\,.\,\vec{c}$ (Distributivity)

(v) $\vec{a}\,.\,\vec{b} = \vec{a}\,.\,\vec{c} \not\Rightarrow \vec{b} = \vec{c}$

Infact $\vec{a}\,.\,\vec{b} = \vec{a}\,.\,\vec{c} \Rightarrow \vec{a}\,.(\vec{b} - \vec{c}) = 0$

$$\vec{a} = 0 \text{ or } \vec{b} = \vec{c} \text{ or } \vec{a}\perp(\vec{b} - \vec{c})$$

(vi) $(\vec{a}\,.\,\vec{b})\,.\,\vec{c}$ is meaningless

(vi) scalar product is not binary operation.

Let $\vec{a}$ and $\vec{b}$ be two vectors such that

$$\vec{a} = \vec{a}_1\,\hat{i} + \vec{a}_2\,\hat{j} + \vec{a}_3\,\hat{k}$$

and $\vec{b} = \vec{b}_1\,\hat{i} + \vec{b}_2\,\hat{j} + \vec{b}_3\,\hat{k}$

Then $\vec{a}\cdot\vec{b} = \vec{a}_1\,\vec{b}_1 + \vec{a}_2\,\vec{b}_2 + \vec{a}_3\,\vec{b}_3$

In particular

$$\vec{a}\cdot\vec{a} = |\vec{a}|^2 = \vec{a}_1^{\,2} + \vec{a}_2^{\,2} + \vec{a}_3^{\,2}$$

For any vector $\vec{a}$,

$$\vec{a} = (\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k}$$

Angle between two vector

(i) If $\vec{a}$ and $\vec{b}$ be two vectors and θ be the angle between them, then

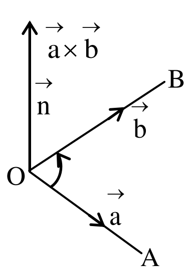
$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\vec{a}}{a}\cdot\frac{\vec{b}}{b} = \hat{a}\cdot\hat{b}$$

(ii) If $\vec{a} = \vec{a}_1\,\hat{i} + \vec{a}_2\,\hat{j} + \vec{a}_3\,\hat{k}$ and

$\vec{b} = \vec{b}_1\,\hat{i} + \vec{b}_2\,\hat{j} + \vec{b}_3\,\hat{k}$ then

$$\cos\theta = \frac{\vec{a}_1\vec{b}_1 + \vec{a}_2\vec{b}_2 + \vec{a}_3\vec{b}_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Vector product



If $\vec{a}$ and $\vec{b}$ be two vectors and θ $(0 \le \theta \le \pi)$ be the angle between them, then their vector (or cross) product is defined to be a vector whose magnitude is ab sin θ

$$\therefore \vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$$

$$= \vec{a}\,\vec{b}\,\sin\theta\,\hat{n}$$

Vector product in terms of components :

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a}\times\vec{b} = (a_2 b_3 - a_3 b_2)\,\hat{i} + (a_3 b_1 - a_1 b_3)\,\hat{j} + (a_1 b_2 - a_2 b_1)\,\hat{k}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Angle between two Vectors :

If θ is the angle between $\vec{a}$ and $\vec{b}$, then

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$$

If $\hat{n}$ is the unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$, then

$$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

(i) The vector product of two parallel vectors is always zero i.e. if vectors a and b are parallel, then $\vec{a}\times\vec{b} = 0$

(ii) If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, then

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\,\hat{n} = \vec{a}\,\vec{b}\,\hat{n}$$

(iii) If $\hat{i}, \hat{j}, \hat{k}$ be three mutually perpendicular unit vectors, then

(a) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

(b) $\hat{i} \times \hat{j} = \hat{k}$, $\hat{i} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{i}$

(c) $\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{j} = -\hat{i}$, $\hat{i} \times \hat{k} = -\hat{j}$

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are any vectors and m,n any scalars then

(i) $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

(Non- commutativity)

but $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

and $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}|$

(ii) $(m\vec{a}) \times \vec{b} = \vec{a} \times (m\vec{b}) = m(\vec{a} \times \vec{b})$

(iii) $(m\vec{a}) \times (n\vec{b}) = (mn)(\vec{a} \times \vec{b})$

(iv) $\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$

(v) $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$

(Distributivity)

(vi) $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \not\Rightarrow \vec{b} = \vec{c}$ Infact

$\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = 0$

$\Rightarrow \vec{a} = 0$ or $\vec{b} = \vec{c}$ or $\vec{a} \parallel (\vec{b} - \vec{c})$

Scalar Triple Product

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors, then their scalar triple product is defined as the dot product of two vectors $\vec{a}$ and $\vec{b} \times \vec{c}$. It is generally denoted by $\vec{a} \cdot (\vec{b} \times \vec{c})$ or $[\vec{a}\ \vec{b}\ \vec{c}]$. It is read as box product of $\vec{a}$, $\vec{b}$, $\vec{c}$. Similarly other scalar triple products can be defined as

$(\vec{b} \times \vec{c}) \cdot \vec{a}$, $(\vec{c} \times \vec{a}) \cdot \vec{b}$.

(i) If $\vec{a} = a1\ell + a2m + a3n$, $\vec{b} = b1\ell + b2m + b3n$ and $\vec{c} = c1\ell + c2m + c3n$, then

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = [\ell mn]$$

(ii) $\vec{a} = \vec{a}_1\hat{i} + \vec{a}_2\hat{j} + \vec{a}_3\hat{k}$, $\vec{b} = \vec{b}_1\hat{i} + \vec{b}_2\hat{j} + \vec{b}_3\hat{k}$ and $\vec{c} = \vec{c}_1\hat{i} + \vec{c}_2\hat{j} + \vec{c}_3\hat{k}$, then

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(iii) For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$

(a) $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

(b) $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}] = 0$

(c) $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]^2$

Properties of Scalar Triple product

(i) The position of (.) and (×) can be interchanged i.e. $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$

but $(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{c} \cdot (\vec{a} \times \vec{b})$

So $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$

(ii) If the cyclic order of vectors is changed, then sign of scalar triple product is changed i.e.

$\vec{a}\,.\,[\vec{b}\times\vec{c}\,] = -\,\vec{a}\,.(\vec{c}\times\vec{b}\,)$

or $[\vec{a}\ \vec{b}\ \vec{c}\,] = -\,[\vec{a}\ \vec{c}\ \vec{b}\,]$

from (i) and (ii) we have

$[\vec{a}\ \vec{b}\ \vec{c}\,] = [\vec{b}\ \vec{c}\ \vec{a}\,] = [\vec{c}\ \vec{a}\ \vec{b}\,]$

$= -\,[\vec{a}\ \vec{c}\ \vec{b}\,] = -\,[\vec{b}\ \vec{a}\ \vec{c}\,] = -\,[\vec{c}\ \vec{b}\ \vec{a}\,]$

(iii) The scalar triple product of three vectors when two of them are equal or parallel, is zero i.e.

$[\vec{a}\ \vec{b}\ \vec{b}\,] = [\vec{a}\ \vec{b}\ \vec{a}\,] = 0$

(iv) The scalar triple product of three mutually perpendicular unit vectors is ±1 Thus

$[\hat{i}\ \hat{j}\ \hat{k}\,] = 1,\ [\hat{i}\ \hat{k}\ \hat{j}\,] = -\,1$

(v) If two of the three vectors $\vec{a}, \vec{b}, \vec{c}$ are parallel then $[\vec{a}\ \vec{b}\ \vec{c}\,] = 0$

(vi) $\vec{a}, \vec{b}, \vec{c}$ are three coplanar vectors if $[\vec{a}\ \vec{b}\ \vec{c}\,] = 0$ i.e. the necessary and sufficient condition for three non-zero collinear vectors to be coplanar is

$[\vec{a}\ \vec{b}\ \vec{c}\,] = 0$

(vii) For any vectors $\vec{a}, \vec{b}, \vec{c}$, d

$[\vec{a} + \vec{b}\ \ \vec{c}\ \ d] = [\vec{a}\ \ \vec{c}\ \ d]$

Vector Triple Product

The vector triple product of three vectors $\vec{a}, \vec{b}, \vec{c}$ is defined as the vector product of two vectors $\vec{a}$ and $\vec{b}\times\vec{c}$. It is denoted by $\vec{a}\times(\vec{b}\times\vec{c}\,)$.

Properties :

(i) Expansion formula for vector triple product is given by

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

$(\vec{b}\times\vec{c})\times\vec{a} = (\vec{b}.\vec{a})\vec{c} - (\vec{c}.\vec{a})\vec{b}$.

(ii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then

$$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_2c_3 - b_3c_2 & b_3c_1 - b_1c_3 & b_1c_2 - b_2c_1 \end{vmatrix}$$

Check Your Progress

1. If ABCDE is a pentagon, then $\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC}$ equals-

(A) 3 $\overrightarrow{AD}$ (B) 3 $\overrightarrow{AC}$

(C) 3 $\overrightarrow{BE}$ (D) 3 $\overrightarrow{CE}$

2. If $\vec{a} = 2\hat{i} + 5\hat{j}$ and $\vec{b} = 2\hat{i} - \hat{j}$, then unit vector in the direction of $\vec{a} + \vec{b}$ is-

(A) $\hat{i} + \hat{j}$ (B) $\sqrt{2}\,(\hat{i} + \hat{j})$

(C) $(\hat{i} + \hat{j})/\sqrt{2}$ (D) $(\hat{i} - \hat{j})/\sqrt{2}$

3. The position vector of a point C with respect to B is $\hat{i} + \hat{j}$ and that of B with respect to A is $\hat{i} - \hat{j}$. The position vector of C with respect to A is-

(A) $2\hat{i}$ (B) $-2\hat{i}$

(C) $2\hat{j}$ (D) $-2\hat{j}$

4. If A, B, C are three points such that $2\overrightarrow{AC} = 3\overrightarrow{CB}$, then $2\overrightarrow{OA} + 3\overrightarrow{OB}$ equals-

(A) $5\overrightarrow{OC}$ (B) $\overrightarrow{OC}$

(C) $-\overrightarrow{OC}$ (D) None of these

5. If the end points of $\overrightarrow{AB}$ are (3, –7) and
(– 1, – 4), then magnitude of $\overrightarrow{AB}$ is-

(A) 2 (B) 3 (C) 4 (D) 5

6. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ then the value of $|\vec{a} + \vec{b}|$ is -

(A) $\sqrt{6}$ (B) $2\sqrt{6}$

(C) $3\sqrt{6}$ (D) $4\sqrt{6}$

7. If vectors $(x - 2)\hat{i} + \hat{j}$ and $(x + 1)\hat{i} + 2\hat{j}$ are collinear, then the value of x is-

(A) 3 (B) 4 (C) 5 (D) 0

8. If points $\hat{i} + 2\hat{k}$, $\hat{j} + \hat{k}$ and $\lambda\hat{i} + \mu\hat{j}$ are collinear, then-

(A) $\lambda = 2, \mu = 1$ (B) $\lambda = 2, \mu = -1$

(C) $\lambda = -1, \mu = 2$ (D) $\lambda = -1, \mu = -2$

9. If $\vec{p} = 2\vec{a} - 3\vec{b}$, $\vec{q} = \vec{a} - 2\vec{b} + \vec{c}$, $\vec{r} = -3\vec{a} + \vec{b} + 2\vec{c}$, $\vec{a}, \vec{b}, \vec{c}$ being non zero, non coplanar vectors then the vectors $-2\vec{a} + 3\vec{b} - \vec{c}$ is equal to -

(A) $\dfrac{-7\vec{q} + \vec{r}}{5}$ (B) $\vec{p} - 4\vec{q}$

(C) $2\vec{p} - 3\vec{q} + \vec{r}$ (D) $4\vec{p} - 2\vec{r}$

10. If the position vectors of four points P, Q, R, S respectively $2\vec{a} + 4\vec{c}$, $5\vec{a} + 3\sqrt{3}\vec{b} + 4\vec{c}$, $-2\sqrt{3}\vec{b} + \vec{c}$ and $2\vec{a} + \vec{c}$ then-

(A) $\overrightarrow{PQ} \parallel \overrightarrow{RS}$ (B) $\overrightarrow{PQ} = \overrightarrow{RS}$

(C) $\overrightarrow{PQ} \neq \overrightarrow{RS}$ (D) None of these

11. If the angle between $\vec{a}$ and $\vec{b}$ is θ then for $\vec{a} \cdot \vec{b}$ □ 0

(A) $0 \le \theta \le \pi$ (B) $0 < \theta < \pi/2$

(C) $\pi/2 \le \theta \le \pi$ (D) $0 \le \theta \le \pi/2$

12. If the moduli of vectors $\vec{a}$ and $\vec{b}$ are 1 and 2 respectively and $\vec{a}\,.\,\vec{b} = 1$, then the angle □ between them is-

(A) $\theta = \pi/6$ (B) $\theta = \pi/3$

(C) $\theta = \pi/2$ (D) $\theta = 2\pi/3$

13. If $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} + 4\hat{k}$, then $(2\vec{a} + \vec{b}).(\vec{a} - 2\vec{b})$ equals-

(A) 14 (B) –14

(C) 0 (D) None of these

14. Angle between the vectors $2\hat{i} + 6\hat{j} + 3\hat{k}$ and $12\hat{i} - 4\hat{j} + 3\hat{k}$ is -

(A) $\cos^{-1}\left(\frac{1}{10}\right)$ (B) $\cos^{-1}\left(\frac{9}{11}\right)$

(C) $\cos^{-1}\left(\frac{9}{91}\right)$ (D) $\cos^{-1}\left(\frac{1}{9}\right)$

15. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + 3\hat{k}$ then $|\vec{a} \times \vec{b}|$ is

(A) $\sqrt{6}$ (B) $2\sqrt{6}$

(C) $\sqrt{70}$ (D) $4\sqrt{6}$

16. If $\vec{a}$ and $\vec{b}$ are two vectors, then-

(A) $|\vec{a} \times \vec{b}| \geq |\vec{a}||\vec{b}|$ (B) $|\vec{a} \times \vec{b}| \leq |\vec{a}||\vec{b}|$

(C) $|\vec{a} \times \vec{b}| > |\vec{a}||\vec{b}|$ (D) $|\vec{a} \times \vec{b}| < |\vec{a}||\vec{b}|$

17. If $[3\hat{i}\;\; 5\hat{j} - 3\hat{k}\;\; \lambda\hat{i} + \hat{k}] = 5$, then the value of □ is-

(A) 1 (B) 2

(C) 3 (D) Not possible

18. If $\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$ & $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$ represent three coterminous edges of a parallelopiped then its volume is-

(A) 60 (B) 15 (C) 30 (D) 40

19. If $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ & $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to-

(A) $20\hat{i} - 3\hat{j} + 7\hat{k}$ (B) $20\hat{i} + 3\hat{j} + 7\hat{k}$

(C) $20\hat{i} + 3\hat{j} - 7\hat{k}$ (D) None of these

20. $\vec{a} \times (\vec{b} \times \vec{c})$ is coplanar with-

(A) $\vec{a}$ and $\vec{b}$ (B) $\vec{b}$ and $\vec{c}$

(C) $\vec{c}$ and $\vec{a}$ (D) None of these

Stretch Yourself

1. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non- coplanar vectors and $\vec{p}$, $\vec{q}$, $\vec{r}$ are vectors defined as

$\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$ then find $(\vec{a}+\vec{b}).\vec{p}+(\vec{b}+\vec{c}).\vec{q}+(\vec{c}+\vec{a}).\vec{r}$

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ be any three non- zero non coplanar vectors and vectors $\vec{p} = \frac{\vec{b}\times\vec{c}}{\vec{a}.\vec{b}\times\vec{c}}$, $\vec{q} = \frac{\vec{c}\times\vec{a}}{\vec{a}.\vec{b}\times\vec{c}}$, $\vec{r} = \frac{\vec{a}\times\vec{b}}{\vec{a}.\vec{b}\times\vec{c}}$, then find$[\vec{p}\ \vec{q}\ \vec{r}]$
3. If $\vec{u} = \vec{a}-\vec{b}$ and $\vec{v} = \vec{a}+\vec{b}$, and $|\vec{a}| = |\vec{b}|$ = 2 then find$|\vec{u}\times\vec{v}|$
4. If $\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$, where $\vec{a}, \vec{b}, \vec{c}$ are non- coplanar vectors, then find $(\vec{a}+\vec{b}+\vec{c}).(\vec{p}+\vec{q}+\vec{r})$
5. If $\vec{a}\ .\ \hat{i}$ = 4, then find $(\vec{a}\times\hat{j}).(2\hat{j}-3\hat{k})$
6. If the vectors $\vec{a} = \hat{i} + a\hat{j} + a^2\hat{k}$, $\vec{b} = \hat{i} + b\hat{j} + b^2\hat{k}$, $\vec{c} = \hat{i} + c\hat{j} + c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$ =0, thenfind the value

Hint to Check your Progress

1 C	2 B	3 C	4 A	5 A
6 D	7 C	8 C	9 C	10 A
11 A	12 D	13 B	14 B	15 C
16 C	17 B	18 D	19 C	20 A