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Linear programming

If a, b, c $\in$R, then the equation ax + by = c is called a **linear equation** in two variables x, y whereas inequalities of the form ax + by $\leq$c, ax + by $\geq$c,
ax + by < c & ax + by > c are called **Linear Inequations** in two variables x & y.

Graph of Linear Equation

Consider a linear inequation ax + by $\leq$c. Drawing the graph of a linear inequation means finding its solution set.

Steps to draw the graph :

To draw the graph of an equation, following procedures are to be made-

(i) Write the inequation ax + by $\leq$ c into an equation
ax + by = c which represent a straight line in xy-plane.

(ii) Put y = 0 in ax + by = c to get point where the line meets x- axis. Similarly, put x = 0 to obtain a point where the line meets y-axis. Join these two points to obtain the graph of the line.

(iii)If the inequation is > or <, then the points lie on this line does not consider and line is drawn dotted or discontinuous.

(iv)If the inequation is $\geq$or $\leq$, then the point lie on the line consider and line is drawn black (bold) or continuous.

(v) This line divides the plane XOY in two region.

To find the region that satisfies the inequation, *we apply the following rules*-

(a) Choose a point [If possible (0, 0)] not lying on this line.

(b) Substitute its coordinates in the inequation. If the inequation is satisfied, then shade the portion of the plane which contains the chosen point, otherwise shade the portion which does not contain this point. The shaded portion represents the solution set.

Feasible Region

The limited (bounded) region of the graph made by two inequations is called **Feasible Region**. All the coordinates of the points in feasible region constitutes the solutions of system of inequations.

The general form of Linear Programming Problems (L.P.P.) is-

Maximize (Minimize) $z = c_1x_1 + c_2x_2 + ...+ c_nx_n$ subjected to

$a_{11}\ x_1 + a_{12}\ x_2\ +......+ a_{1n}\ x_n$ {$\leq$, =, $\geq$b$_1$

$a_{21}\ x_1 + a_{22}\ x_2 + ...+ a_{2n}\ x_n$ {$\leq$,, =, $\geq$} b_2

....

$a_{m1}\, x_1 + a_{m2}\, x_2 + \ldots\ldots + a_{mn}\, x_n \{\leq,, =, \geq\}\, b_n$

and $x_1, x_2, x_3, \ldots\ldots, x_n \geq 0$

where $x_1, x_2, x_3, \ldots x_n$ are the variables whose values are to be determined and are called the **decision variables**. The inequation are called **constraints** and the function to be maximized or minimized is called the **objective function**.

Some Definitions

(i) Solution : A set of values of the decision variables which satisfy the constraints of a Linear Programming Problem (L.P.P.) is called a solution of the L.P.P.

(ii) Feasible Solution : A solution of L.P.P. which also satisfy the non- negative restrictions of the problem is called the feasible solution.

(iii) Optimal Solution : A feasible solution which maximize or minimize i.e. which optimize the objective function of L.P.P. called an optimal solution.

(iv) Iso-Profit Line: The line is drawn in geometrical area of feasible region of L.P.P. for which the objective function remains constant at all the points lie on this line, is called iso-profit line.

Graphical Method of Solution of Linear Programming Problem

The graphical method for solving linear programming problems is applicable to those problems which involve only two variables. This method is based upon a theorem, called **extreme point theorem**, which is stated as follows-

Extreme Point Theorem: If a L.P.P. admits an optimal solution, then at least one of the extreme (or corner) points of the feasible region gives the optimal solution.

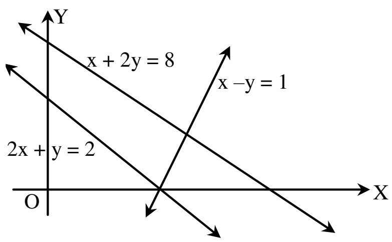
(i) Find the solution set of the system of simultaneous linear inequations given by constraints and non-negativity restrictions.

(ii) Find the coordinates of each of corner points of the feasible region.

(iii) Find the values of the objective function at each of the corner points of the feasible region. By the extreme point theorem one of the corner points will provide the optimal value of the objective function. The coordinates of that corner point determine the optimal solution of the L.P.P.

Check Your Progress

1. For the following shaded area, the linear constraints except $x \geq 0$ and $y \geq 0$, are -



(A) $2x + y \leq 2, x - y \leq 1, x + 2y \leq 8$

(B) $2x + y \geq 2, x - y \leq 1, x + 2y \leq 8$

(C) $2x + y \geq 2, x - y \geq 1, x + 2y \leq 8$

(D) $2x + y \geq 2, x - y \geq 1, x + 2y > 8$

2. The solution set of constraints $x + 2y \geq 11$, $3x + 4y \leq 30$, $2x + 5y \leq 30$, $x \geq 0$, $y \geq 0$ includes the point -

(A) (2, 3) (B) (3, 2)

(C) (3, 4) (D) (4, 3)

3. The region represented by the inequation system $x, y \geq 0$, $y \leq 6$, $x + y \leq 3$, is-

(A) unbounded in first quadrant

(B) unbounded in first and second quadrants

(C) bounded in first quadrant

(D) None of these

4. In equations $3x - y \geq 3$ and $4x - y > 4$-

(A) have solution for positive x and y

(B) have no solution to positive x and y

(C) have solution for all x

(D) have solution for all y

5. The graph of $x \leq 2$ and $y \geq 2$ will be situated in the -

(A) first and second quadrant

(B) second and third quadrant

(C) first and third quadrant

(D) third and fourth quadrant

6. The solution set of linear constraints $x - 2y \geq 0$, $2x - y \leq -2$ and $x, y \geq 0$, is-

(A) $\left(-\frac{4}{3}, -\frac{2}{3}\right)$ (B) (1, 1)

(C) $\left(0, \frac{2}{3}\right)$ (D) (0, 2)

7. A vertex of a feasible region by the linear constraints $3x + 4y \leq 18$, $2x + 3y \geq 3$ and $x, y \geq 0$, is-

   (A) (0, 2) (B) (4.8, 0)

   (C) (0, 3) (D) None of these

8. In which quadrant, the bounded region for equations $x + y \leq 1$ and $x - y \leq 1$ is situated-

   (A) I, II

   (B) I, III

   (C) II, III

   (D) All the four quadrants

9. The necessary condition for third quadrant region in x – y plane, is-

   (A) $x > 0, y < 0$ (B) $x < 0, y < 0$

   (C) $x < 0, y > 0$ (D) $x < 0, y = 0$

10. The graph of inequations $x \leq y$ and $y > x + 3$ located in -

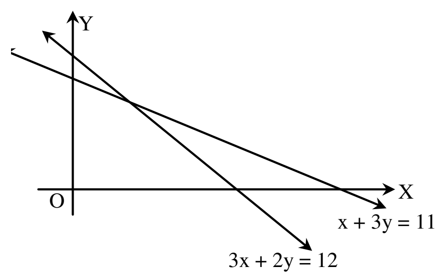
    (A) II quadrant

    (B) I, II quadrants

    (C) I, II, III quadrants

    (D) II, III, IV quadrants

11. For the following feasible region, the linear constraints are-



    (A) $x \geq 0, y \geq 0, 3x + 2y \geq 12, x + 3y \geq 11$

    (B) $x \geq 0, y \geq 0, 3x + 2y \leq 12, x + 3y \geq 11$

    (C) $x \geq 0, y \geq 0, 3x + 2y \leq 12, x + 3y \leq 11$

    (D) None of these

12. The region represented by $2x + 3y - 5 \leq 0$ and $4x - 3y + 2 \leq 0$, is-

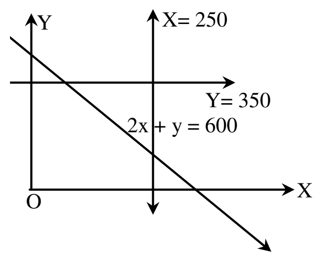
    (A) not in first quadrant

    (B) bounded in first quadrant

(C) unbounded in first quadrant

(D) None of these

13. For the following feasible region, the linear constraints except $x \geq 0$ and $y \geq 0$, are -



(A) $x \geq 250, y \leq 350, 2x + y = 600$

(B) $x \leq 250, y \leq 350, 2x + y = 600$

(C) $x \leq 250, y \leq 350, 2x + y \geq 600$

(D) $x \leq 250, y \leq 350, 2x + y \leq 600$

14. For the L.P. problem Max $z = 3x_1 + 2x_2$ such that $2x_1 - x_2 \geq 2$, $x_1 + 2x_2 \leq 8$ and $x_1, x_2 \geq 0$, then z =

(A) 12 (B) 24

(C) 36 (D) 40

15. The L.P. problem Max $z = x_1 + x_2$ such that $-2x_1 + x_2 \leq 1$, $x_1 \leq 2$, $x_1 + x_2 \leq 3$ and $x_1, x_2 \geq 0$ has -

(A) one solution

(B) three solution

(C) an infinite number of solutions

(D) None of these

Stretch Yourself

1. For the L.P. problem Min. $z = -x_1 + 2x_2$ such that $-x_1 + 3x_2 \leq 0$, $x_1 + x_2 \leq 6$, $x_1 - x_2 \leq 2$ and $x_1, x_2 \geq 0$,find x_1
2. Find the maximum value of $Z = 4x + 2y$ subjected to the constraints $2x + 3y \leq 18$, $x + y \geq 10$; $x, y \geq 0$,
3. For the L.P. problem find Min $z = 2x_1 + 3x_2$ such that $-x_1 + 2x_2 \leq 4$, $x_1 + x_2 \leq 6$, $x_1 + 3x_2 > 9$ and $x_1, x_2 \geq 0$
4. For the L.P.problem Min $z = x_1 + x_2$ such that $5x_1 + 10x_2 \leq 0$, $x_1 + x_2 \geq 1$, $x_2 \leq 4$ and $x_1, x_2 \geq 0$ Find the nature of solution
5. Find the minimum value of $P = 6x + 16y$ subject to constraints $x \leq 40$, $y \geq 20$ and $x, y \geq 0$ is-

Hint to Check Your Progress

1 B 2 C 3 C 4 A 5 A

6 A 7 D 8 D 9 B 10 C

11 A 12 B 13 D 14 B 15 C