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Mathematical Reasoning

In reasoning we communicate our ideas or thoughts with the help of sentences in a particular language.

"A sentence is called a mathematically acceptable statement if it is either true or false but not both".

A statement is assumed to be either true or false. A true statement is known as a valid statement and a false statement is known as an invalid statement.

Negation of Statement

The denial of a statement p is called its negation and is written as ~ p, and read as 'not p'.

Negation of any statement p is formed by writing "It is not the case that ....."

or

"It is false that....."

or

inserting the word "not" in p.

(1) Negation : If p and q are two statements then

$$\boxed{\sim (p \to q) = p \wedge \sim q}$$

(2) Contrapositive : If p and q are two statements, then the contrapositive of the implication

$$\boxed{p \to q = (\sim q) \to (\sim p)}$$

Compound Statement

If a statement is combination of two or more statements, then it is said to be a compound statement.

And each statement which form a compound statement are known as its sub-statements or component statements.

Basic connectives :

In the compound statement, we have learnt that the words **'or'** & **'and** connect two or more statements. These are called connectives. When we use these compound statements, it is necessary to understand the role of these words.

The word "AND" : Any two statements can be connected by the word "and" to form a compound statement.

Rule - (1) The compound statement with word "and" is true if all its component statements are true.

Rule - (2) The compound statement with word "and" is false if any or all of its component statements are false.

Conditional Statement

If p and q are any two statement then the compound statement in the form "If p then q" is called a conditional statement or an implication.

The statement "If p then q" is denoted by

$p \to q$ or $p \Rightarrow q$ (to be read as p implies q)

In the implication "$p \rightarrow q$", p is called the antecedent (or the hypothesis) and q the consequent (or the conclusion)

Tautology and Fallacy

(a) Tautology : This is a statement which always true for all truth values of its components.

b) Fallacy (contradiction) : This is statement which is always false for all truth values of its components.

Algebra of Statement

Statements satisfy many laws some of which are given below -

(1) Idempotent Laws : If p is any statement then

(i) $p \vee p \equiv p$

(ii) $p \wedge p \equiv p$

(2) Associative Laws : If p, q, r are any three statements, then

(i) $p \vee (q \vee r) = (p \vee q) \vee r$

(ii) $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

(3) Commutative Laws : If p, q are any two statements, then

(i) $p \vee q = q \vee p$ (ii) $p \wedge q = q \wedge p$

(4) Distributive Laws : If p, q, r are any three statements, then

(i) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

(ii) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(5) Identity Laws : If p is any statement, t is tautology and c is a contradiction, then

(i) $p \vee t = t$ (ii) $p \wedge t = p$

(iii) $p \vee c = p$ (iv) $p \wedge c = c$

(6) Complement Laws : If t is a tautology, c is a contradiction and p is any statement, then

(i) $p \vee (\sim p) = t$ (ii) $p \wedge (\sim p) = c$

(iii) $\sim t = c$ (iv) $\sim c = t$

(7) Involution law : If p is any statement, then $\sim(\sim p) = p$

(8) De morgan's law : If p and q are two statements, then

(i) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

(ii) $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

Stretch Yourself

1- Check the following sentences are statement give reason for your answer .

(a) There is no rain without clouds.

(b) Tajmahal is the most beautiful building of the world.

(c) Every function is a relation.

2- Write the negation of the following statement:

(a) All primes are even

(b) Every integer is greater than Zero.

3- Identify the component statements of the following compound statement .

(a) The sky is blue and the grass is green.

(b) All rational number are real and all real number are complex.

4- Check the pair of statements negation of each other :
   (a) The number x is a relation number.
   (b) The number x is an irrational number.
5- Write the component statements and check the compound statement is true or false.
   (a) 59 is divisible by 3 and 5 .
   (b) All living things have two eyes and two legs .
6- Write the truth value of the following statements :
(a) New Delhi is in India or 2+2=5
(b) New Delhi is in America or 2+2=5

7- Identify the quantifier and write the negation of each of the following statements :
   (a) All English teachers are female .
   (b) There exist a real number, whose square is not positive.
8- Check whether the following pair of statements are negations of each other: give reasons for your answer
   (a) X+Y = Y+X is true for every real number X and Y .
   (b) There exists real number X and Y for which X+Y = Y+X .
9- Write the composite and converse of following statements :
(a) If P is a prime number, then P is odd .
(b) If the two lines are parallels, then they do not intersect in the same plane.
10- Prove $\sqrt{19}$ is not a rational number.