312en13

13

SIMPLE HARMONIC MOTION

You are now familiar with motion in a straight line, projectile motion and circular motion. These are defined by the path followed by the moving object. But some objects execute motion which are repeated after a certain interval of time. For example, beating of heart, the motion of the hands of a clock, to and fro motion of the swing and that of the pendulum of a bob are localised in space and repetitive in nature. Such a motion is called periodic motion. It is universal phenomenon.

In this lesson, you will study about the periodic motion, particularly the oscillatory motion which we come across in daily life. You will also learn about simple harmonic motion. Wave phenomena – types of waves and their characteristics– form the subject matter of the next lesson.

OBJECTIVES

After studying this lesson, you should be able to :

- *show that an oscillatory motion is periodic but a periodic motion may not be necessarily oscillatory;*
- *define simple harmonic motion and represent it as projection of uniform circular motion on the diameter of a circle;*
- *derive expressions of time period of a given harmonic oscillator;*
- *derive expressions for the potential and kinetic energies of a simple harmoic oscillator; and*
- *distinguish between free, damped and forced oscillations.*

13.1 PERIODIC MOTION

You may have observed a clock and noticed that the pointed end of its seconds hand and that of its minutes hand move around in a circle, each with a fixed period. The seconds hand completes its journey around the dial in one minute but the minutes hand takes one hour to complete one round trip. However, a pendulum bob moves to and fro about a mean position and completes its motion from one end to the other and back to its initial position in a fixed time. A motion which repeats itself after a fixed interval of time is called ***periodic motion***. There are two types of periodic motion : (i) ***non–oscillatory,*** and (ii) ***oscillatory***. The motion of the hands of the clock is non-oscillatory but the to and fro motion of the pendulum bob is oscillatory. However, both the motions are periodic. It is important to note that an oscillatory motion is normally periodic but a periodic motion is not necessarily oscillatory. Remember that a motion which repeats itself in equal intervals of time is periodic and if it is about a mean position, it is ***oscillatory***.

We know that earth completes its rotation about its own axis in 24 hours and days and nights are formed. It also revolves around the sun and completes its revolution in 365 days. This motion produces a sequence of seasons. Similarly all the planets move around the Sun in elliptical orbits and each completes its revolution in a fixed interval of time. These are examples of periodic non-oscillatory motion.

Jean Baptiste Joseph Fourier
(1768 – 1830)

French Mathematician, best known for his Fourier series to analyse a complex oscillation in the form of series of sine and consine functions.

Fourier studied the mathematical theory of heat conduction.
He established the partial differential equation governing heat diffusion and solved it by using infinite series of trigonometric functions.

Born as the ninth child from the second wife of a taylor, he was orphened at the age of 10. From the training as a priest, to a teacher, a revolutionary, a mathematician and an advisor to Nepolean Bonapart, his life had many shades.

He was a contemporary of Laplace, Lagrange, Biot, Poission, Malus, Delambre, Arago and Carnot. Lunar crator Fourier and his name on Eiffel tower are tributes to his contributions.

ACTIVITY 13.1

Suppose that the displacement y of a particle, executing simple harmonic motion, is represented by the equation :

$$y = a \sin \theta \qquad (13.1)$$

or

$$y = a \cos \theta \qquad (13.2)$$

From your book of mathematics, obtain the values of sin θ and cos θ for θ = 0, 30^0, 60^0, 90^0, 120^0, 150^0, 180^0, 240^0, 300^0, 330^0 and 360^0. Then assuming that a = 2.5cm, determine the values of y corresponding to each angle using the relation $y = a \sin\theta$. Choose a suitable scale and plot a graph between y and θ. Similarly, using the relation $y = a \cos\theta$, plot another graph between y and θ. You will note that both graphs represents an oscillation between $+a$ and $-a$. It shows that a certain type of oscillatory motion can be represented by an expression containing sine or cosine of an angle or by a combination of such expressions.

13.1.1 Displacement as a Function of Time

Periodic Motion

When an object repeats its motion after a definite interval of time, its motion is said to be periodic.

Let the position of an object change from O to B, from B to O; then from O to A and finally from A to O, after a fixed interval of time T.

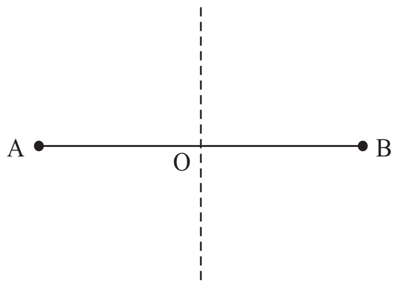


Fig. 13.1

Then, the changes in the position or displacement of the object can be expressed as a function of time:

$$x = af(t + T)$$

where a is a constant and T is the time after which the value of x is repeated

For each time interval T:

$$x = af(T) = 0 \text{ at } t = 0$$

$$x = af(T + T/4) = a \text{ at } t = T/4$$

$$x = af\left(T+\frac{T}{2}\right) = 0 \text{ at } t = \frac{T}{2}$$

$$x = af\left(T+\frac{3T}{4}\right) = -a \text{ at } t = \frac{3T}{4}$$

$$x = af(T + T) = 0 \text{ at } t = T$$

....

....

Thus, x is function of t and it repeats its motion after an interval T. Hence, the motion is periodic.

Now check your progress by answering the following questions.

INTEXT QUESTIONS 13.1

1. What is the difference between a periodic motion and an oscillatory motion?
2. Which of the following examples represent a periodic motion?
   (i) A bullet fired from a gun,
   (ii) An electron revolving round the nucleus in an atom
   (iii) A vehicle moving with a uniform speed on a road
   (iv) A comet moving around the Sun, and
   (v) Motion of an oscillating mercury column in a U-tube.
3. Give an example of (i) an oscillatory periodic motion and (ii)Non-oscillatory periodic motion.

13.2 SIMPLE HARMONIC MOTION : CIRCLE OF REFERENCE

The oscillations of a harmonic oscillator can be represented by terms containing sine and cosine of an angle. If the displacement of an oscillatory particle from its mean position can be represented by an equation $y = a \sin\theta$ or $y = a \cos\theta$ or $y = A \sin\theta + B \cos\theta$, where a, A and B are constants, the particle executes simple harmonic motion. We define simple harmonic motion as under :

A particle is said to execute simple harmonic motion if it moves to and fro about a fixed point periodically, under the action of a force* F *which is directly proportional to its displacement* x *from the fixed point and the direction of the force is opposite to that of the displacement. We shall restrict our discussion to linear oscillations. Mathematically, we express it as

$$F = -kx$$

where k is constant of proportionality.

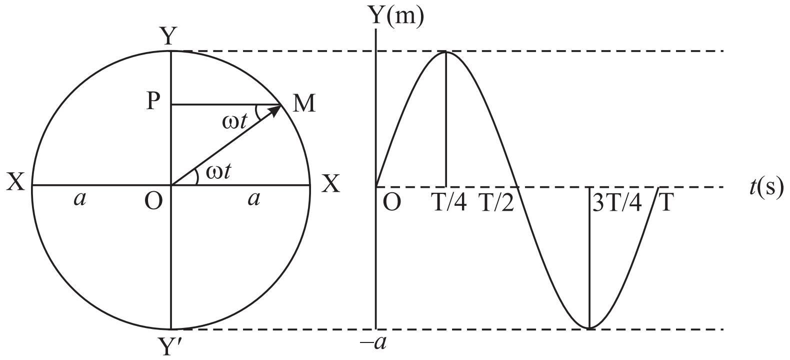


Fig. 13.2 : Simple harmonic motion of P is along YOY′

To derive the equation of simple harmonic motion, let us consider a point M moving with a constant speed υ in a circle of radius a (Fig. 13.2) with centre O. At $t = 0$, let the point be at X. The position vector OM specifies the position of the moving point at time t,. It is obvious that the position vector OM, also called the ***phaser,*** rotates with a constant angular velocity $\omega = \upsilon / a$. The acceleration of the point M is $\upsilon^2/a = a\,\omega^2$ towards the centre O. At time t, the component of this acceleration along OY = $a\omega^2 \sin \omega t$. Let us draw MP perpendicular to YOY′. Then P can be regarded as a particle of mass m moving with an acceleration $a\omega^2 \sin \omega t$. The force on the particle P towards O is therefore given by

$$F = ma\omega^2 \sin \omega t$$

But $\sin \omega t = y/a$. *Therefore*

$$F = m\omega^2 y \tag{13.3}$$

The displacement is measured from O towards P and force is directed towards O. Therefore,

$$F = -m\omega^2 y$$

Since this force is directed towards O, and is proportional to displacement 'y' of P from O. we can say that the particle P is executing simple harmonic motion.

Let us put $m\omega^2 = k$, a constant. Then Eqn. (13.3) takes the form

$$F = -k\,y \tag{13.4}$$

The constant k, which is force per unit displacement, is called ***force constant***. The angular frequency of oscillations is given by

$$\omega^2 = k / m \tag{13.5}$$

In one complete rotation, OM describes an angle 2π and it takes time T to complete one rotation. Hence

$$\omega = 2\pi/T \tag{13.6}$$

On combining Eqns. (13.5) and (13.6), we get an expression for time period :

$$T = 2\pi\sqrt{k/m} \tag{13.7}$$

This is the time taken by P to move from O to Y, then through O to Y′ and back to O. During this time, the particle moves once on the circle and the foot of perpendicular from its position is said to make an oscillation about O as shown in Fig.13.1.

Let us now define the basic terms used to describe simple harmonic motion.

13.2.1 Basic Terms Associated with SHM

Displacement is the distance of the harmonic oscillator from its mean (or equilibrium) position at a given instant.

Amplitude is the maximum displacement of the oscillator on either side of its mean position.

Time period is the time taken by the oscillator to complete one oscillation. In Fig. 13.1, OP, and OY respectively denote displacement and amplitude.

Frequency is the number of oscillations completed by an oscillator in one second. It is denoted by v. The SI unit of frequency is hertz (symbol Hz). Since v is the number of oscillations per second, the time taken to complete one oscillation is $1/v$. Hence $T = 1/v$ or $v = (1/T)\ \text{s}^{-1}$. As harmonic oscillations can be represented by expressions containing $\sin\theta$ and or $\cos\theta$, we introduce two more important terms.

Phase ϕ is the angle whose sine or cosine at a given instant indicates the position and direction of motion of the oscillator. It is expressed in radians.

Angular Frequency ω describes the rate of change of phase angle. It is expressed in radian per second. Since phase angle ϕ changes from 0 to 2π radians in one complete oscillation, the rate of change of phase angle is $\omega = 2\pi/T = 2\pi\,v$ or $\omega = 2\pi v$.

Example 13.1 : A tray of mass 9 kg is supported by a spring of force constant k as shown in Fig. 13.3. The tray is pressed slightly downward and then released. It begins to execute SHM of period 1.0 s. When a block of mass M is placed on the tray, the period increases to 2.0 s. Calculate the mass of the block.

Solution: The angular frequency of the system is given by $\omega = \sqrt{k/m}$, where m is the mass of the oscillatory system. Since $\omega = 2\pi/T$, from Eqn. (13.7) we get

$$4\pi^2/T^2 = \frac{k}{m}$$

or

$$m = \frac{kT^2}{4\pi^2}$$

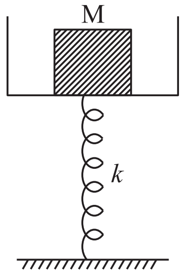


Fig. 13.3

When the tray is empty, $m = 9$ kg and $T = 1$ s. Therefore

$$9 = \frac{k(1)^2}{4\pi^2}$$

On placing the block, $m = 9 + M$ and $T = 2$ s. Therefore, $9 + M = k \times (2)^2/4\pi^2$

From the above two equations we get

$$\frac{(9+M)}{9} = 4$$

Therefore, $M = 27$ kg.

Example 13.2 : A spring of force constant 1600 N m^{-1} is mounted on a horizontal table as shown in Fig. 13.4. A mass $m = 4.0$ kg attached to the free end of the spring is pulled horizontally towards the right through a distance of 4.0 cm and then set free. Calculate (i) the frequency (ii) maximum acceleration and (iii) maximum speed of the mass.

Solution : $\omega = \sqrt{k/m} = \sqrt{1600/4}$

$= 20$ rad s^{-1}.

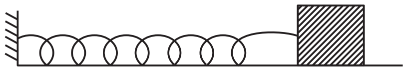


Fig. 13.4

Therefore $\nu = 20/2\pi = 3.18$ Hz. Maximum acceleration $= a\,\omega^2 = 0.04 \times 400 = 16$ m s^{-2}, and $\upsilon_{max} = a\,\omega = 0.04 \times 20 = 0.8$ m s^{-1}.

13.3 EXAMPLES OF SHM

In order to clarify the concept of SHM, some very common examples are given below.

13.3.1 Horizontal Oscillations of a Spring-Mass System

Consider a elastic spring of force constant k placed on a smooth horizontal surface and attached to a block P of mass m. The other end of the spring is attached to a rigid wall (Fig. 13.5)). Suppose that the mass of the spring is negligible in comparison to the mass of the block.

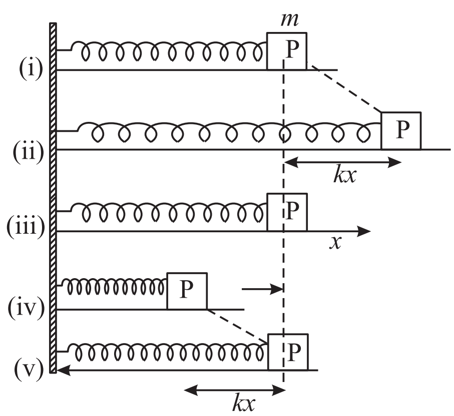


Fig.13.5 : Oscillations of a spring-mass system

Let us suppose that there is no loss of energy due to air resistance and friction. We choose *x–axis* along the horizontal direction. Initially, that is, at $t = 0$, the block is at rest and the spring is in relaxed condition [Fig.13.5(i)]. It is then pulled horizontally through a small distance [Fig. 13.5 (ii)]. As the spring undergoes an extension x, it exerts a force kx on the block. The force is directed against the extension and tends to restore the block to its equilibrium position. As the block returns to its initial position [Fig. 13.5 (iii)], it acquires a velocity υ and hence a kinetic energy $K = (1/2)\, m\, \upsilon^2$. Owing to inertia of motion, the block overshoots the mean position and continues moving towards the left till it arrives at the position shown in Fig. 13.5 (iv). In this position, the block again experiences a force kx which tries to bring it back to the initial position [Fig. 13.5 v]. In this way, the block continues oscillating about the mean position. The time period of oscillation is $2\pi\sqrt{m/k}$, where k is the force per unit extension of the spring.

13.3.2 Vertical Oscillations of a Spring–Mass System

Let us suspend a spring of force constant k from a rigid support [Fig.13.6(a)]. Then let us attach a block of mass m to the free end of the spring. As a result of this, the spring undergoes an extension, say l [Fig.13.6(b)]. Obviously, the force constant of the spring is $k = mg/l$. Let us now pull down the block through a small distance, y (Fig.13.6 (c)]. A force ky acts on the block vertically upwards. Therefore, on releasing the block, the force ky pulls it upwards. As the block returns to its initial position, it continues moving upwards on account of the velocity it has gained. It overshoots the equilibrium position by a distance y. The compressed spring now applies on it a restoring force downwards. The block moves downwards and again overshoots the equilibrium position by almost the same vertical distance y. Thus, the system continues to execute vertical oscillations. The angular frequency of vertical oscillations is

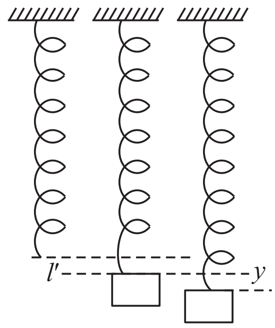


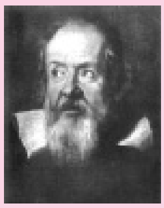
Fig. 13.6: Vertical oscillations of a a spring–mass system

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

Hence $$T = 2\pi\sqrt{\frac{m}{k}} \qquad (13.8)$$

This result shows that acceleration due to gravity does not influence vertical oscillations of a spring–mass system.

Galileo Galilei
(1564-1642)



Son of Vincenzio Galilei, a wool merchant in Pisa, Italy, Galileo is credited for initiating the age of reason and experimentation in modern science. As a child, he was interested in music, art and toy making. As a young man, he wanted to become a doctor. To pursue the study of medicine, he entered the University of Pisa. It was here that he made his first discovery - the isochronosity of a pendulum, which led Christian Huygen to construct first pendulum clock.

For lack of money, Galileo could not complete his studies, but through his efforts, he learnt and developed the subject of mechanics to a level that the Grand Duke of Tuscany appointed him professor of mathematics at the University of Pisa.

Galileo constructed and used telescope to study celestial objects. Through his observations, he became convinced that Copernican theory of heliocentric universe was correct. He published his convincing arguments in the form of a book, "A Dialogue On The Two Principal Systems of The World", in the year 1632. The proposition being at variance with the Aristotelian theory of geocentric universe, supported by the Church, Galileo was prosecuted and had to apologize. But in 1636, he published another book "Dialogue On Two New Sciences" in which he again showed the fallacy in Aristotle's laws of motion.

Because sophisticated measuring devices were not available in Galileo's time, he had to apply his ingenuity to perform his experiments. He introduced the idea of thought-experiments, which is being used even by modern scientists, in spite of all their sophisticated devices.

13.3.3 Simple Pendulum

A simple pendulum is a small spherical bob suspended by a long cotton thread held between the two halves of a clamped split cork in a stand,as shown in Fig. 13.7. The bob is considered a point mass and the string is taken to be *inextensible*. The Pendulum can oscillate freely about the point of suspension.

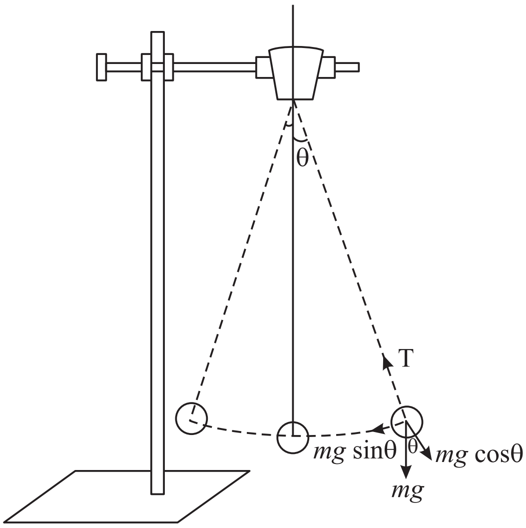


Fig.13.7 : Simple Pendulum

When the pendulum is displaced through a *small distance* from its equilibrium position and then let free, it executes angular oscillations in a vertical plane about its equilibrium position. The distance between the point of suspension and the centre of gravity of the bob defines the length of the pendulum. The forces acting on the bob of the pendulum in the displaced position shown in Fig. 13.7 are : (*i*) the weight of the bob $m\mathbf{g}$ vertically downwards, and (*ii*) tension in the string $\mathbf{T}$ acting upwards along the string.

The weight mg is resolved in two components : (a) $mg \cos\theta$ along the string but opposite to $\mathbf{T}$ and (b) $mg \sin\theta$ perpendicular to the string. The component mg

$\cos\theta$ balances the tension **T** and the component $mg\sin\theta$ produces acceleration in the bob in the direction of the mean position. The restoring force, therefore, is $mg\sin\theta$. For small displacement x of the bob, the restoring force is $F = mg\theta = mg\,x/l$. The force per unit displacement $k = mg/l$ and hence

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/l}{m}} = \sqrt{\frac{g}{l}}$$

or

$$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

Hence,

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (13.9)$$

Measuring Weight using a Spring

We use a spring balance to measure weight of a body. It is based on the assumption that within a certain limit of load, there is equal extension for equal load, i.e., load/extension remains constant (force constant). Therefore, extension varies linearly with load. Thus you can attach a linear scale alongside the spring and calibrate it for known load values. The balance so prepared can be used to measure unknown weights.

Will such a balance work in a gravity free space, as in a space-rocket or in a satellite? Obviously not becuase in the absence of gravity, no extension occurs in the spring. Then how do they measure mass of astronauts during regular health check up? It is again a spring balance based on a different principle. The astronaut sits on a special chair with a spring attached to each side (Fig.13.8). The time period of oscillations of the chair with and without the astronaut is determined with the help of an electronic clock :

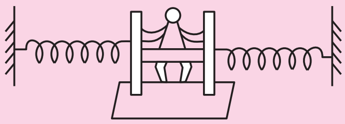


Fig. 13.8 : Spring balance for measuring the mass of an astronaut

$$T_1^2 = \frac{4\pi^2 m}{k}$$

where m is mass of the astronaut. If m_0 is mass of the chair, we can write

$$T_0^2 = \frac{4\pi^2 m_0}{k}$$

T_1 is time period of oscillation of the chair with the astronaut and T_0 without the astronaut.

On subtracting one from another, we get

$$T_1^2 - T_0^2 = \frac{4\pi^2}{k}(m - m_o)$$

$$\Rightarrow \quad m = \frac{k}{4\pi^2}(T_1^2 - T_0^2) + m_o$$

Because the values of T_0 and k are fixed and known, a measure of T_1 itself shows the variation in mass.

Example 13.3 : Fig. 13.9 shows an oscillatory system comprising two blocks of masses m_1 and m_2 joined by a massless spring of spring constant k. The blocks are pulled apart, each with a force of magnitude F and then released. Calculate the angular frequency of each mass. Assume that the blocks move on a smooth horizontal plane.

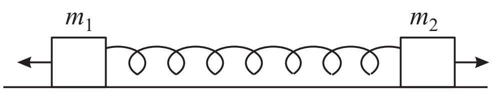


Fig. 13.9 : Oscillatory system of masses attached to a spring

Solution : Let x_1 and x_2 be the displacements of the blocks when pulled apart. The extension produced in the spring is $x_1 + x_2$. Thus the acceleration of m_1 is $k(x_1 + x_2)/m_1$ and acceleration of m_2 is $k(x_1 + x_2)/m_2$. Since the same spring provides the restoring force to each mass, hence the net acceleration of the system comprising of the two masses and the massless spring equals the sum of the acceleration produced in the two masses. Thus the acceleration of the system is

$$a = \frac{k(x_1 + x_2)}{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)} = \frac{kx}{\mu}$$

where $x = x_1 + x_2$ is the extension of the spring and μ is the reduced mass of the system. The angular frequency of each mass of the system is therefore,

$$\omega = \sqrt{k/\mu} \qquad (13.10)$$

Such as analysis helps us to understand the vibrations of diatomic molecules like H_2, Cl_2, HCl, etc.

INTEXT QUESTIONS 13.2

1. A small spherical ball of mass m is placed in contact with the sunface on a smooth spherical bowl of radius r a little away from the bottom point. Calculate the time period of oscillations of the ball (Fig. 13.10).

2. A cylinder of mass m floats vertically in a liquid of density ρ. The length of the cylinder inside the liquid is l. Obtain an expression for the time period of its oscillations (Fig. 13.11).

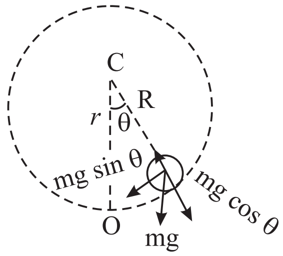


Fig. 13.10

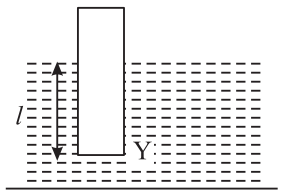


Fig.13.11

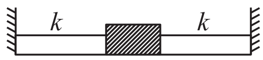


Fig. 13.12

3. Calculate the frequency of oscillation of the mass m connected to two rubber bands as shown in Fig. 13.12. The force constant of each band is k. (Fig. 13.12)

13.4 ENERGY OF SIMPLE HARMONIC OSCILLATOR

As you have seen, simple harmonic motion can be represented by the equation

$$y = a \sin \omega t \tag{13.11}$$

When t changes to $\mathbf{t} + \Delta t$, y changes to $y + \Delta y$. Therefore, we can write

$$y + \Delta y = a \sin\omega (t + \Delta t) = a \sin (\omega t + \omega \Delta t)$$

$$= a [\sin\omega t \cos \omega\Delta t + \cos\omega t \sin \omega\Delta t]$$

As $\Delta t \to 0$, $\cos \omega\Delta t \to 1$ and $\sin \omega \Delta t \to \omega \Delta t$. Then

$$y + \Delta y = a \sin \omega t + a \omega\Delta t \cos \omega t. \tag{13.12}$$

Subtracting Eqn. (13.11) from Eqn. (13.12), we get

$$\Delta y = \Delta t \omega a \cos \omega t$$

so that $\quad \Delta y/\Delta t = \omega a \cos \omega t$

or $\quad \upsilon = \omega a \text{ cost } \omega t \quad (13.13)$

where $\upsilon = \Delta y/\Delta t$ is the velocity of the oscillator at time t. Hence, the ***kinetic energy*** of the oscillator at that instant of time is

$$K = (1/2)\, m\upsilon^2 = (1/2)\, \omega^2 a^2 \cos^2 \omega t \tag{13.14}$$

Let us now calculate the potential energy of the oscillator at that time. When the displacement is y, the restoring force is ky, where k is the force constant. For this purpose we shall plot a graph of restoring force ky versus the displacement y. We get a straight line graph as shown in Fig. 13.13. Let us take two points P and Q and drop perpendiculars PM and QN on x–axis. As points P and Q are close to each other, trapezium PQNM can be regarded as a rectangle. The area of this rectangular strip is ($ky\ \Delta y$). This area equals the work done against the restoring force ky when the displacement changes by a small amount Δy. The area of the triangle OBC is, therefore, equal to the work done in the time displacement changes from O to OB (= y) = $\frac{1}{2}ky^2$. This work done against the conservative force is the ***potential energy*** U of the oscillator. Thus, the potential energy of the oscillator when the displacement is y is

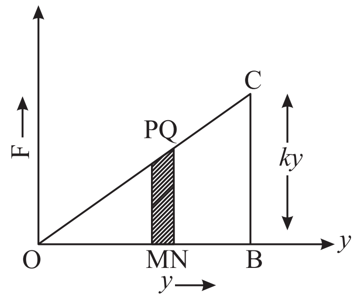


Fig.13.13 : Graph between the displacement y and the restoring force ky

$$U = \frac{1}{2}ky^2$$

But $\omega^2 = k/m$. Therefore, substituting $k = m\omega^2$ in above expression we get

$$U = \frac{1}{2}m\omega^2 y^2$$

Further as $y = a \sin \omega t$, we can write

$$U = \frac{1}{2}m\omega^2 a^2 \sin^2\omega t \qquad (13.15)$$

On combining this result with Eqn. (13.14), we find that total energy of the oscillator at any instant is given by

$$E = U + K$$

$$= \frac{1}{2}m\omega^2 a^2 (\sin^2\omega t + \cos^2\omega t)$$

$$= \frac{1}{2}ma^2\omega^2 \qquad (13.16)$$

The graph of kinetic energy K, potential energy U and the total energy E versus displacement y is shown in Fig.13.14. From the graph it is evident that for $y = 0$, $K = E$ and $U = 0$. As the displacement y from the mean position increases, the kinetic energy decreases but potential energy increases. At the mean position, the potential energy is zero but kinetic energy is maximum. At the extreme positions, the energy is wholly potential. However, the sum $K + U = E$ is constant.

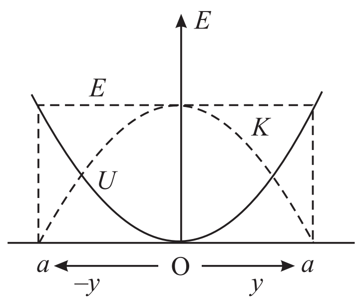


Fig.13.14 : Variation of potential energy U, kinetic energy K, and total energy E with displacement from equilibrium position

INTEXT QUESTIONS 13.3

1. Is the kinetic energy of a harmonic oscillator maximum at its equilibrium position or at the maximum displacement position? Where is its acceleration maximum?
2. Why does the amplitude of a simple pendulum decrease with time? What happens to the energy of the pendulum when its amplitude decreases?

13.5 DAMPED HARMONIC OSCILLATIONS

Every oscillating system normally has a viscous medium surrounding it. As a result in each oscillation some of its energy is dissipated as heat. As the energy of oscillation decreases the amplitude of oscillation also decreases. The amplitude of oscillations of a pendulum in air decreases continuously. Such oscillations are called damped oscillations. To understand **damped oscillations** perform activity 13.2.

Activity 13.2

Take a simple harmonic oscillator comprising a metal block B suspended from a fixed support S by a spring G. (Fig. 13.15(a). Place a tall glass cylinder filled two thirds with water, so that the block is about 6 cm below the surface of water and about the same distance above the bottom of the beaker. Paste a millimetre scale (vertically) on the side of the cylinder just opposite the pointer attached to the block. Push the block a few centimetres downwards and then release it. After each oscillation, note down the uppermost position of the pointer on the millimetre scale and the time. Then plot a graph between time and the amplitude of oscillations. Does the graph [Fig. 13.15 (b)] show that the amplitude decreases with time. Such oscillations are said to be ***damped oscillations***.

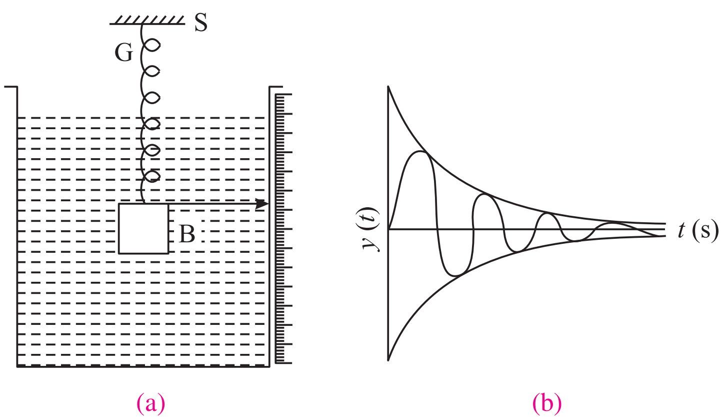


Fig. 13.15 : Damped vibrations : (a) experimental setup; (b) graphical representation

Notes

13.6 FREE AND FORCED VIBRATIONS : RESONANCE

To understand the difference between these phenomena, let us perform the following activity :

ACTIVITY 13.3

Take a rigid horizontal rod fixed at both ends. Tie a loose but strong thread and hang the four pendulums A,B,C,D, as shown in Fig. 13.16. The pendulums A and B are of equal lengths, whereas C has a shorter and D has a longer length than A and B. The pendulum B has a heavy bob. Set pendulum B into oscillations. You will observe that after a few minutes, the other three pendulums also begin to oscillate. (It means that if a no. of oscillators are coupled, they transfer their energy. This has an extremely important implication for wave propagation.) You will note that the amplitude of A is larger. Why? Each pendulum is an oscillatory system with natural frequency of its own. The pendulum B, which has a heavy bob, transmits its vibrations to each of the pendulums A, C and D. As a consequence, the pendulums C and D are forced to oscillate *not with their respective natural* frequency *but with the frequency of the pendulum B.* The phenomenon is called ***forced oscillation.*** By holding the bob of any one of these pendulums, you can force it to oscillate with the frequency of C or of D. Both C and D are forced to oscillate with the frequency of B. However, pendulum A on which too the oscillations of the pendulums B are impressed, oscillates with a relatively large amplitude with ***its natural frequency.*** This phenomenon is known as **resonance.**

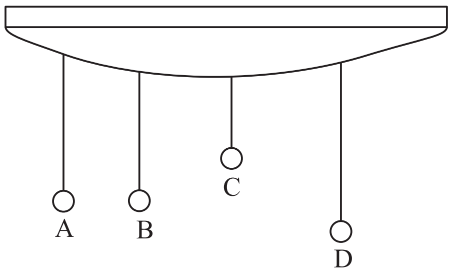


Fig. 13.16: Vibrations and resonance.

When the moving part of an oscillatory system is displaced from its equilibrium position and then set free, it oscillates to and fro about its equilibrium position with a frequency that depends on certain parameters of the system only. Such oscillations are known as ***free vibrations.*** The frequency with which the system oscillates is known as ***natural frequency.*** When a body oscillates under the influence of an external periodic force, the oscillations are called ***forced oscillations.*** In forced oscillations, the body ultimately oscillates with the frequency of the external force. The oscillatory system on which the oscillations are impressed is called driven and the system which applies the oscillating force is known as the driver. The particular case of forced oscillations in which natural frequencies of the driver and the driven are equal is known as ***resonance.*** In resonant oscillations, the driver and the driven reinforce each other's oscillations and hence their amplitudes are maximum.

INTEXT QUESTIONS 13.4

1. When the stem of a vibrating tuning fork is pressed against the top of a table, a loud sound is heard. Does this observation demonstrate the phenomenon of resonance or forced vibrations? Give reasons for your answer. What is the cause of the loud sound produced?

2. Why are certain musical instruments provided with hollow sound boards or sound boxes?

Mysterious happenings and resonance

1. Tacoma Narrows Suspension Bridge, Washington, USA collapsed during a storm within six months of its opening in 1940. The wind blowing in gusts had frequency equal to the natural frequency of the bridge. So it swayed the bridge with increasing amplitude. Ultimately a stage was reached where the structure was over stressed and it collapsed.

   The events of suspension bridge collapse also happened when groups of marching soldiers crossed them. That is why, now, the soldiers are ordered to break steps while crossing a bridge.

   The factory chimneys and cooling towers set into oscillations by the wind and sometimes get collapsed.

2. You might have heard about some singers with mysterious powers. Actually, no such power exists. When they sing, the glasses of the window panes in the auditorium are broken. They just sing the note which matches the natural frequency of the window panes.

3. You might have wondered how you catch a particular station you are interested in by operating the tuner of your radio or TV? The tuner in fact, is an electronic oscillator with a provision of changing its frequency. When the frequency of the tuner matches the frequency transmitted by the specific station, resonance occurs and the antenna catches the programme broadcasted by that station.

WHAT YOU HAVE LEARNT

- Periodic motion is a motion which repeats itself after equal intervals of time.
- Oscillatory motion is to and fro motion about a mean position. An oscillatory motion is normally periodic but a periodic motion may not *necessarily be oscillatory*.

- Simple harmonic motion is to and fro motion under the action of a restoring force, which is proportional to the displacement of the particle from its equilibrium position and is always directed towards the mean position.
- Time period is the time taken by a particle to complete one oscillation.
- Frequency is the number of vibrations completed by the oscillator in one second.
- Phase angle is the angle whose sine or cosine at the given instant indicates the position and direction of motion of the particle.
- Angular frequency is the rate of change of phase angle. Note that $\omega = 2\pi/T = 2\pi v$ where ω is the angular frequency in rads^{-1}, v is the frequency in hertz (symbol : Hz) and T is the time period in seconds.
- Equation of simple harmonic motion is

$$y = a \sin (\omega t + \phi_0)$$

or

$$y = a \cos (\omega t + \phi_0)$$

where y is the displacement from the mean position at a time, ϕ_0 is the initial phase angle (at $t = 0$).

- When an oscillatroy system vibrates on its own, its vibrations are said to be free. If, however, an oscillatory system is driven by an external system, its vibrations are said to be forced vibrations. And if the frequency of the driver equals to the natural frequency of the driven, the phenomenon of resonance is said to occur.

TERMINAL EXERCISE

1. Distinguish between a periodic and an oscillatory motion.
2. What is simple harmonic motion?
3. Which of the following functions represent (i) a simple harmonic motion (ii) a periodic but not simple harmonic (iii) a non periodic motion? Give the period of each periodic motion.

   (1) $\sin \omega t + \cos \omega t$ (2) $1 + \omega^2 + \omega t$

   (3) $3 \cos (\omega t - \frac{\pi}{4})$
4. The time period of oscillations of mass 0.1 kg suspended from a Hooke's spring is 1s. Calculate the time period of oscillation of mass 0.9 kg when suspended from the same spring.
5. What is phase angle? How is it related to angular frequency?

6. Why is the time period of a simple pendulum independent of the mass of the bob, when the period of a simple harmonic oscillator is $T = 2\pi\sqrt{m/k}$?

7. When is the magnitude of acceleration of a particle executing simple harmonic motion maximum? When is the restoring force maximum?

8. Show that simple harmonic motion is the projection of a uniform circular motion on a diameter of the circle. Obtain an expression for the time period of a simple harmonic oscillator in terms of mass and force constant.

9. Obtain expressions for the instantaneous kinetic energy potential energy and the total energy of a simple harmonic oscillator.

10. Show graphically how the potential energy U, the kinetic energy K and the total energy E of a simple harmonic oscillator vary with the displacement from equilibrium position.

11. The displacement of a moving particle from a fixed point at any instant is given by $x = a \cos \omega t + b \sin \omega t$. Is the motion of the particle simple harmonic? If your answer is no, explain why? If your answer is yes, calculate the amplitude of vibration and the phase angle.

12. A simple pendulum oscillates with amplitude 0.04 m. If its time period is 10 s, calculate the maximum velocity.

13. Imagine a ball dropped in a frictionless tunnel cut across the earth through its centre. Obtain an expression for its time period in terms of radius of the earth and the acceleration due to gravity.

14. Fig. 13.17 shows a block of mass $m = 2$ kg connected to two springs, each of force constant $k = 400\ \mathrm{N\ m^{-1}}$. The block is displaced by 0.05 m from equilibrium position and then released. Calculate (a) The angular frequency ω of the block, (b) its maximum speed; (c) its maximum acceleration; and total energy dissipated against damping when it comes to rest.

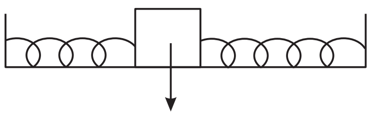
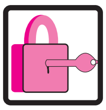


Fig.13.17



ANSWERS TO INTEXT QUESTIONS

13.1

1. A motion which repeats itself after some fixed interval of time is a periodic motion. A to and fro motion on the same path is an oscillatory motion. A periodic motion may or may not be oscillatory but oscillation motion is perodic.

2. (ii), (iv), (v);

3. (i) To and fro motion of a pendulum.

   (ii) Motion of a planet in its orbit.

13.2

1. Return force on the ball when displaced a distance x from the equilibrium position is $mg \sin \theta = mg\, \theta = mg\, x/r$. $\therefore\ \omega = \sqrt{g/r}$.

2. On being pushed down through a distance y, the cylinder experiences an upthrust $y\alpha\rho g$. Therefore $\omega^2 = \dfrac{\alpha\rho g}{m}$ and $m = \alpha p\rho$. From the law of flotation m = mass of black. Hence, $\omega^2 = g/l$ or $T = 2\pi\ \sqrt{l/g}$.

3. $\omega^2 = k/m$ and hence $v = 1/2\pi\sqrt{k/m}$. Note that when the mass is displaced, only one of the bands exerts the restoring force.

13.3

1. K.E is maximum at mean position or equilibrium position; acceleration is maximum when displacement is maximum.

2. As the pendulum oscillates it does work against the viscous resistance of air and friction at the support from which it is suspended. This work done is dissipated as heat. As a consequence the amplitude decreases.

13.4

1. When an oscillatory system called the driver applies is periodic of force on another oscillatory system called the driven and the second system is forced to oscillate with the frequency of the first, the phenomenon is known as forced vibrations. In the particular case of forced vibrations in which the frequency of the driver equals the frequency of the driven system, the phenomenon is known as resonance.

2. The table top is forced to vibrate not with its natural frequency but with the frequency of the tuning fork. Therefore, this observation demonstrates forced vibrations. Since a large area is set into vibrations, the intensity of the sound increases.

3. The sound board or box is forced to vibrate with the frequency of the note produced by the instrument. Since a large area is set into vibrations, the intensity of the note produced increases and its duration decreases.

Answers to Terminal Problems

4. 3s

11. $A = \sqrt{a^2 + b^2}, \theta = \tan^{-1}\left(\frac{a}{b}\right)$

12. $\frac{2}{\pi} \times 10^{-3}$ m s^{-1}

14. (*a*) 14.14 s^{-1}

    (*b*) 0.6 m s^{-1}

    (*c*) 0.3 m s^{-2}

    (*d*) 0.5 J